DERIVATIVE APPROXIMATION FOR ANALOG DESIGN

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ABSTRACT

This paper presents a technique of derivative approximation in the design of analog circuits, which can be useful in the implementation of a function derivative. We present an example and discuss some aspects involved in the application of this technique.

1. INTRODUCTION

Analog circuit design is an important aspect in many applications, such as artificial neural networks, fuzzy systems, and as a support for digital circuits [1]. In the design of analog circuits we can employ several techniques and mathematical resources to obtain the implementation of a certain function. For example, at the device level, the MOS transistor can be exploited in the various regions of operation. In saturation, it operates as a quadratic function; in subthreshold, as an exponential, and in triode, as a resistor. At the circuit level, functions such as addition, subtraction, multiplication, derivative and integral can be exploited. This work presents a derivative approximation in the design of analog circuits. This technique has been successfully employed in the design of radial basis circuits [2]-[4]. We present an example and discuss some aspects involved in the application of the technique.

2. DERIVATIVE APPROXIMATION

Consider the expression of the derivative *f'(x)* of a function *f(x)*:

$$f'(x) = \lim_{\Delta \to 0} \frac{f(x+\Delta) - f(x)}{\Delta} \tag{1}$$

We can rearrange (1) in the following way:

$$f'(x) \cong \frac{1}{\Delta}\left[f(x+\Delta) - f(x)\right] + \varepsilon \tag{2}$$

where ε is an error term and 1/Δ is an amplitude gain. Equation (2) can be applied to any differentiable function.

We note that the derivative function can be approximated by a difference operation with a shift in inputs. The approximation will be closer to the derivative function the smaller the shift. Furthermore, if we eliminate the factor 1/Δ and the term ε, we conclude that the function *g(x)* given by:

$$g(x) = [\ f(x+\Delta) - f(x)\ ] \tag{3}$$

has the form of the derivative function except by a gain factor 1/Δ and an error term ε.

3. EXAMPLE: A HYPERBOLIC SECANT SQUARED

The differential pair is one of the most used blocks in analog circuits. We will present the application of the derivative approximation technique in the design of a circuit that implements the transconductance function of an emitter-coupled pair (Fig 1). The differential current *Id* and the differential voltage *Vd* are related by:

$$Id = \alpha_F . I_t . \tanh\left(Vd / 2Vt\right) \tag{4}$$

where I_t is the tail current, α_F is the relation I_C / I_E and *Vt* is the thermal voltage. The transconductance g_m is given by equation (5):

$$g_m = \frac{\partial Id}{\partial Vd} = \frac{\alpha_F . I_t}{2Vt} . \operatorname{sech}^2\left(\frac{Vd}{2Vt}\right) \tag{5}$$

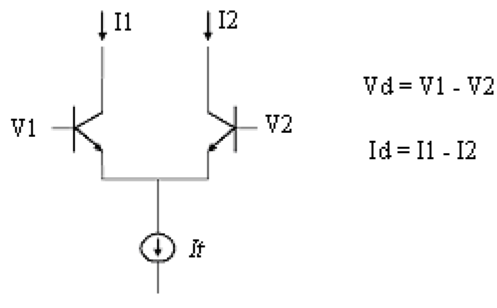

Fig. 1: Emitter-coupled pair.

The output of the differential pair is a hyperbolic tangent function (Fig. 2). The transconductance curve is its first derivative and has the form of a hyperbolic secant squared (Fig. 3).

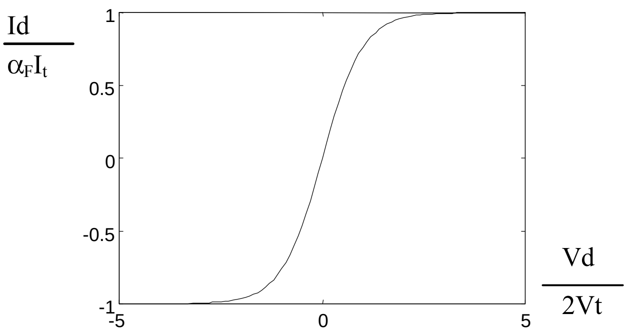

Fig. 2: DC Transfer Characteristic of an Emitter-Coupled Pair.

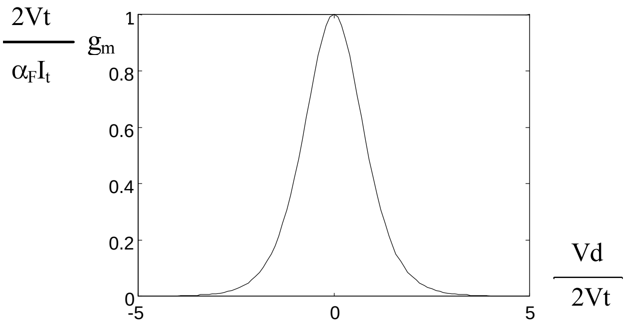

Fig. 3: DC Transconductance Characteristic of an Emitter-Coupled Pair.

The circuit that realizes the derivative function is presented in Fig. 4 (see appendix). The experimental data presented in this paper were obtained with a breadboard prototype. The differential pairs were implemented with OTAs presented in the LM13600 dual-OTA integrated circuit. The output voltage Vout is obtained by the difference of OTA1 and OTA2 outputs which is implemented by current-mode summing at the resistor Ro. Ib1 and Ib2 allow the amplitude adjustment. The currents are in the milliamp range. The voltage shift vdelta in the input of OTA2 is fixed in 100mV. OTA3 is a multiplier and allows width adjustment by Ib3 and center adjustment by Vc.

4. RESULTS

Fig. 5 shows the experimental results for the hyperbolic secant squared implementation. Fig. 6 shows the circuit response for triangular signal in the input.

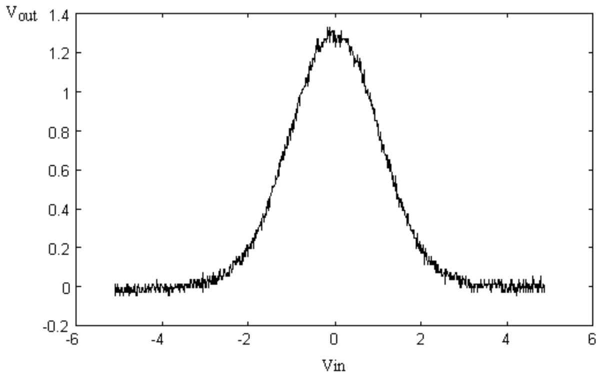

Fig. 5: Experimental data obtained for the circuit.

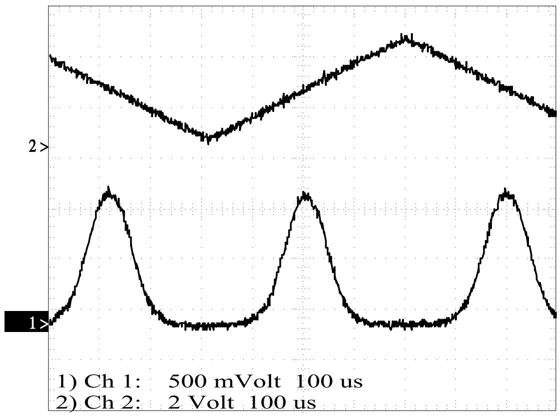

Fig. 6: Circuit response for a triangular input signal.

5. CONCLUSIONS

The proposed technique allows the realization of the derivative of a circuit response. An example of application where this technique could be useful is in the implementation of functions for artificial neural networks with on-chip learning. Finally, the technique can be a part of the repertoire available to the designer in the implementation of analog functions.

6. ACKNOWLEDGMENTS

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7. REFERENCES

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8. APPENDIX

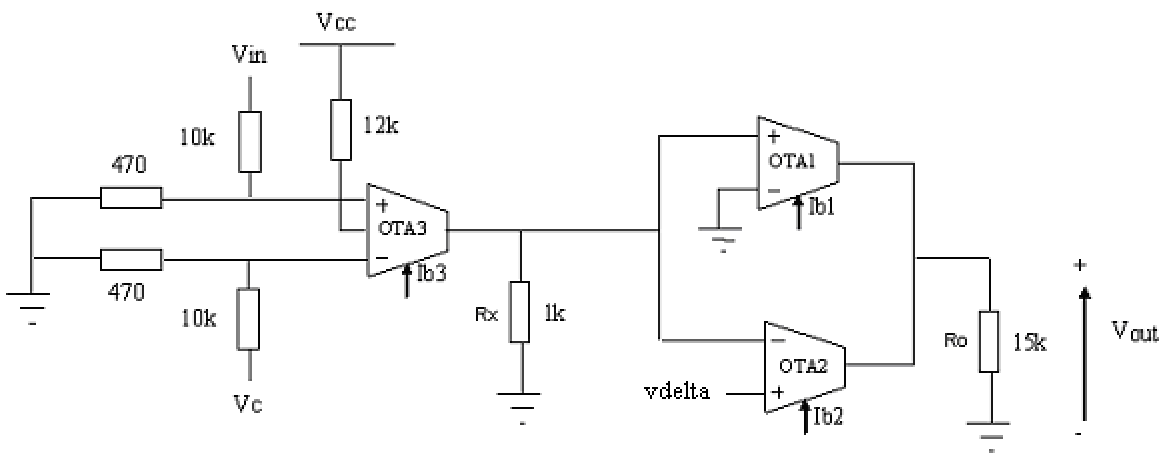

Fig. 4: The Hyperbolic Secant Squared circuit.