Behavioral Modeling of Dual-band Radio Frequency Power Amplifiers using Volterra Series

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ABSTRACT

In this article, the band-pass characteristic of dual-band power amplifiers is explored to reduce the number of coefficients of the Volterra series, without any deterioration in its modeling accuracy. In here, it is reported the mathematical development needed to simplify the Volterra series, which can be seen as an extension for the dual-band case of a previous work that addressed the single band case. The condition to be satisfied by the carrier frequencies to guarantee the validity of the simplified model is identified. Matlab simulation results from two examples are presented to validate the study.

Keywords

Power amplifiers; radio frequency; Volterra series; wireless communication systems.

1. INTRODUCTION

In wireless communication systems, a dual-band power amplifier (DBPA) [1] is excited by the signal

$$in = \operatorname{Re}\{x_1 \exp(j2\pi f_1 t)\} + \operatorname{Re}\{x_2 \exp(j2\pi f_2 t)\}$$
(1)

where x_1 and x_2 are the complex-valued envelopes that modulate the carriers of frequencies f_1 and f_2 , respectively. According to Cripps [2], independent of the class of operation of the transistors present in the DBPA, there is a compromise between linearity and efficiency. To be efficient in the conversion from DC supply power to radio frequency (RF) output power, DBPAs must operate at high power levels. At this situation, the presence of nonlinearities due to gain compression is evident, which causes spectral spreading in frequencies around the carriers (called intermodulation distortions) and also at harmonic frequencies of the carriers. Matching networks are designed to attenuate the harmonic distortions. This way, the DBPA output signal is described by

$$out = \operatorname{Re}\{y_1 \exp(j2\pi f_1 t)\} + \operatorname{Re}\{y_2 \exp(j2\pi f_2 t)\}$$
(2)

where y_1 and y_2 are the complex-valued envelopes that modulate the carriers of frequencies f_1 and f_2 , respectively. However, it is not possible to attenuate the intermodulation distortions through a simple linear filtering. As consequence, these distortions cause interferences between users allocated at adjacent channels. To avoid the intermodulation distortions, it is enough to decrease the average power of the DBPA [2]. But, in this case, the DBPA efficiency will be decreased as well, which lowers the time of battery autonomy in handsets and raises the cost of the base station operation, associated with energy consumption and acquisition/maintenance of equipment for heating dissipation [3].

To get high efficiency and high linearity simultaneously, an excellent cost-benefit alternative is the technique known as digital baseband pre-distortion (DPD) [4]. The DPD consists of a cascade connection of two systems (the DBPA being one of them) that have transfer characteristics which are inverse with respect to each other. This way, the input and output signals in the cascade connection are identical, even when each block alone has nonlinear behavior. Therefore, in a DPD scheme, the DBPA is connected in cascade with its inverse system. Moreover, the inverse system is always placed before the DBPA, because the power levels are much lower at the DBPA input.

From Amin *et al* [5], the successful design of a DPD for the linearization of amplifiers with more than one carrier depends on the availability of a low cost model that can estimate with high accuracy the complex-valued envelopes at the DBPA output based on the complex-valued envelopes applied at its input. A feed-forward discrete-time low-pass equivalent behavioral model adequate for DPD purposes is the Volterra series [6]. The contribution of this work is twofold. First, in here the technique introduced in [7] for reducing the number of parameters of a Volterra series, intended to the low-pass equivalent behavioral modeling of single band PAs, is extended to the case of DBPAs. Second, it is shown the condition that must be fulfilled by the two carrier frequencies in order to keep the accuracy of the Volterra series.

This work is organized as follows. Section 2 addresses the theoretical development of this work. Section 3 reports the simulation results to validate the contributions of this work. Conclusions are given in Section 4.

2. VOLTERRA SERIES

Created by the Italian Vito Volterra, the Volterra series is capable of representing nonlinear systems with memory [6]. The nonlinear effects are considered through polynomial approaches and the memory effects are obtained by formulating the instantaneous output (n) as a function not only of the instantaneous input value, but also of the past input values. A Volterra series has adjustable parameters, called kernels. The Volterra series is described by

$$y_{a}(n) = \sum_{P=1}^{P_{0}} \sum_{P_{1}=0}^{P} \cdots \sum_{P_{4}=0}^{P} \sum_{q_{1}=0}^{M} \cdots \sum_{q_{R}=q_{R-1}}^{M} \sum_{q_{R+1}=0}^{M} \cdots$$

$$\sum_{\substack{q_{R+P_{2}} \\ = \\ q_{P-1} \\ p_{1}=P_{2}-1}}^{M} \sum_{\substack{q_{R+P_{2}+1}=0 \\ = \\ q_{P+1}=P_{1}-1}}^{M} \sum_{q_{R+P_{2}+P_{3}}=0}^{M} \sum_{q_{R+P_{2}+P_{3}}}^{M} \sum_{q_{R+P_{2}+P_{3}+1}=0}^{M} \cdots$$

$$\sum_{\substack{q_{P}=q_{P-1} \\ q_{1}=P_{2}-1}}^{M} h_{P_{1},P_{2},P_{3},P_{4}}, \prod_{j=1}^{P} x_{1}(n-q_{j_{1}})$$

$$\prod_{j_{2}=P_{1}+1}^{P+P_{2}} x_{1}^{*}(n-q_{j_{2}}) \prod_{j_{3}=P_{1}+P_{2}+1}^{P_{1}+P_{3}} x_{2}(n-q_{j_{3}})$$

$$\prod_{j_{4}=P_{1}+P_{2}+P_{3}+1}^{P_{1}+P_{2}+P_{3}+1} x_{2}^{*}(n-q_{j_{4}})$$
(3)

where P_0 is the polynomial order truncation and M is the memory length. Also, x_a^* represents the complex conjugate of x_a and aassumes the values 1 and 2. In (3), the symmetric coefficients were excluded to attain a more compact model. Even with this, the number of coefficients of (3) grows exponentially with P_0 and M.

An amplifier is a band-pass system because in the signals handled by it there are spectral components only at frequencies nearby the carriers. For single band amplifiers, Benedetto *et al.* [7] showed that the great majority of the contributions of a Volterra series are contributions in harmonic frequencies of the carrier and, therefore, only a small subset of contributions is important to describe the behavior of the amplifier around the carrier.

At this article, aiming to reduce the quantity of coefficients of (3), the proposed strategy in [7] is extended to the case of dual-band amplifiers. With this, it is observed that there are necessary, to model y_1 , only the terms of (3) where the number of parcels x_2 is equal to the number of parcels x_2^* and also where the number of parcels x_1 is equal to the number of the parcels x_1^* plus 1. This way, (3) gets reduced to

$$y_{1}(n) = \sum_{p=1}^{P} \sum_{\substack{P=1 \ P_{1}=1}}^{P} \sum_{q_{1}=0}^{M} \cdots \sum_{\substack{q_{p_{1}}=q_{p_{1}-1} \ q_{p_{1}+1}=0}}^{M} \cdots \sum_{\substack{q_{2p_{1}-1}=q_{2p_{1}-2} \ q_{2p_{1}-2}=0}}^{M} \sum_{\substack{q_{1}+p=0 \ q_{p_{1}+p}=0}}^{M} \cdots \sum_{\substack{q_{2p_{1}-2}=q_{2p_{2}-2} \ q_{p_{1}+p_{2}-2}}}^{M} \prod_{j_{1}=1}^{M} \sum_{q_{1}+p=0}^{M} \cdots \sum_{\substack{q_{2p_{1}-2}=q_{2p_{2}-2} \ q_{p_{1}+p_{2}-2}}}^{M} \prod_{j_{1}=1}^{M} \sum_{q_{1}+p=0}^{M} \cdots \sum_{j_{1}=1}^{M} \sum_{q_{1}+p=0}^{M} \sum_{j_{1}=1}^{M} \sum_{q_{1}+p=0}^{M} \cdots \sum_{j_{1}=1}^{M} \sum_{q_{1}+p=0}^{M} \sum_{q_{$$

where $P_0 = 2P-1$. To model y_2 just change the roles of x_1 and x_2 in (4).

As expected, (4) drastically reduces the number of coefficients of (3). However, to not have any loss in modeling accuracy, it is necessary that:

$$rf_1 \neq sf_2 \tag{5}$$

for any integers r and s between 1 and P_0 .

3. VALIDATION

The Volterra series from (3) and (4) are now analyzed for $P_0 = 5$ and M = 1, where (3) has 1286 coefficients and (4) has 70 coefficients. For this analysis, in Matlab software, the Volterra series of (3) and (4) are applied to fit input-output data obtained from a DBPA represented by a Wiener architecture, which utilizes a finite impulse response (FIR) filter with two coefficients and a fifth order polynomial. The complex-valued envelopes that modulate the carriers f_1 and f_2 are, respectively, a 3GPP WCDMA signal with a bandwidth of 8.84 MHz and an LTE OFDMA signal with a bandwidth of 10 MHz. The complex-valued envelopes were captured at a sampling frequency of 61.44 MHz. A total of 5000 samples were collected. The least squares algorithm used 2500 samples to extract the Volterra kernels, leaving the other 2500 samples to the modeling validation. Error signals defined by the difference between desired and estimated output signals were calculated. The normalized mean square error (NMSE), according to [8], was then evaluated.

3.1 Case 1: f_1 and f_2 comply with (5)

Here, $f_1 = 0.9$ GHz and $f_2 = 2.5$ GHz. Table 1 shows the NMSE results.

Table 1. NMSE results in dB					
Band	Model	Extraction	Validation		
f_1	(3)	-73.0	-34.9		
f_1	(4)	-61.7	-61.4		
f_2	(3)	-74.9	-37.2		
f_2	(4)	-63.0	-59.5		

Note that, for the validation data set and for both f_1 and f_2 , the errors obtained from (4) were much smaller than the errors obtained from (3). In fact, (3) contemplates all of the 70 contributions of (4) and, thus, during the extraction (3) provides smaller errors than (4). However, the 1216 additional terms from (3) only serve to model measurement noises (associated to contributions at harmonic frequencies of f_1 and f_2) and to render the regression matrix of the least squares algorithm (LSA) practically singular, which significantly deteriorates the quality of the identified coefficients. Figures 1 and 2 illustrate the transfer characteristics in both f_1 and f_2 , respectively. Specifically, Figs. 1 and 2 show the normalized output amplitude as a function of the normalized input amplitude measured at the DBPA and estimated by (4). Observe that, for both bands, excellent estimations are provided by the Volterra series from (4).



Figure 1. Output amplitude as a function of the input amplitude for the WCDMA signal: measured at the DBPA and estimated by the Volterra series of (4).



Figure 2. Output amplitude as a function of the input amplitude for the OFDMA signal: measured at the DBPA and estimated by the Volterra series of (4).

3.2 Case 2: f_1 and f_2 do not comply with (5) Here, $f_1 = 1.25$ GHz and $f_2 = 2.5$ GHz. Therefore, the condition described by (5) is not satisfied. For example, for r = 2 and s = 1, $rf_1 = sf_2$. Table 2 shows the NMSE results.

Table 2. NMSE results in dB					
Band	Model	Extraction	Validation		
f_1	(3)	-75.4	-36.9		
f_1	(4)	-26.2	-23.4		
f_1	Mod. ^(*)	-63.9	-62.9		
f_2	(3)	-79.0	-39.6		
f_2	(4)	-28.5	-26.3		
f_2	Mod. (*)	-66.2	-61.0		

(*) Equation (4) including additional terms

Observe that the Volterra series of (3) did not present satisfactory results for the validation set due to the ill conditioning of the regression matrix of the LSA. However, the Volterra series of (4) also did not show accurate results, not even for the extraction data set. In fact, in the case 2, (5) is not satisfied. Therefore, terms that involve $x_2x_1^*$, $x_2x_1x_1^*x_1^*$, $x_1x_1x_1x_2^*$, $x_2x_2x_2^*x_1^*$ and $x_2x_2x_1^*x_1^*x_1^*$ must be added to (4) that estimates the output at band f_1 , increasing to 118 the number of coefficients. In a similar way, terms that involve x_1x_1 , $x_2x_2x_1^*x_1^*$, $x_2x_1x_1x_2^*$, $x_1x_1x_1^*x_1$ and $x_1x_1x_1x_1x_2^*$ must be added to (4) that estimates the output at band f_2 , increasing the number of coefficients to 112. Table 2 shows that, with these alterations made, the obtained NMSE results are excellent. Figures 3 and 4 illustrate the transfer characteristics in both f_1 and f_2 , respectively, measured on the DBPA and simulated by the original Volterra series of (4) (designated by simulated 1) and by the modified Volterra series of (4) that includes the aforementioned additional terms (designated by simulated 2). Observe that almost perfect estimations are achieved by the modified Volterra series that includes additional terms.



Figure 3. Output amplitude as a function of the input amplitude for the WCDMA signal: measured at the DBPA and estimated by the original Volterra series of (4) and by the modified Volterra series of (4) that includes additional terms.



Figure 4. Output amplitude as a function of the input amplitude for the OFDMA signal: measured at the DBPA and estimated by the original Volterra series of (4) and by the modified Volterra series of (4) that includes additional terms. Figure 5 and 6 show the power spectral densities (PSDs) of the error signals for the WCDMA and OFDMA bands, respectively, obtained when using the original Volterra series of (4) (designated by simulated error 1) and when using the modified Volterra series of (4) that includes the aforementioned additional terms (designated by simulated error 2). Observe that the inclusion of the additional terms has dramatically reduced the PSD of the error signals at the whole displayed spectral range.



Figure 5. PSDs of the error signals for the WCDMA signal: simulated error 1 uses the original Volterra series of (4); simulated error 2 uses the modified Volterra series of (4) that includes additional terms.



Figure 6. PSDs of the error signals for the OFDMA signal: simulated error 1 uses the original Volterra series of (4); simulated error 2 uses the modified Volterra series of (4) that includes additional terms.

4. CONCLUSIONS

Using the Volterra series for the modeling of DBPAs on a total blind manner, in addition to require a hugely computational complexity, may result in inaccurate models due to the ill conditioning of the regression matrix of the LSA. Therefore, by exploring the band-pass characteristic of the DBPAs, the number of coefficients of a Volterra series can be drastically reduced. Yet, the relation between f_1 and f_2 must be considered, because, depending on the scenario, additional terms must mandatorily be included on the Volterra series to keep its high accuracy.

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6. REFERENCES

- A. Cidronali, N. Giovannelli, I. Magrini, and G.Manes, "Compact concurrent dual-band power amplifier for 1.9 GHzWCDMA and 3.5 GHz OFDM wireless systems," in *Proc. 38th Eur. Microw. Conf.*, 2008, pp. 1545–1548.
- [2] S. Cripps, *RF Power Amplifiers for Wireless Communications*. Norwood, MA: Artech House, 2006.
- [3] F. H. Raab, P. Asbeck, S. Cripps, P. B. Kenington, Z. B. Popovic, N. Pothecary, J. F. Sevic, and N. O. Sokal, "Power amplifiers and transmitters for RF and microwave," *IEEE Trans. Microw. Theory Tech.*, vol.50, no.3, pp.814–826, Mar. 2002.
- [4] P. B. Kenington, *High Linearity RF Amplifier Design*. Norwood, MA: Artech House, 2000.
- [5] S. Amin, P. N. Landin, P. Handel, and D. Ronnow, "Behavioral Modeling and Linearization of Crosstalk and Memory Effects in RF MIMO Transmitters," *IEEE Trans. Microw. Theory Tech.*, vol.62, no.4, pp.810–823, Apr. 2014.
- [6] V. Mathews and G. Sicuranza, *Polynomial Signal Processing*. New York: Wiley, 2000.
- [7] S. Benedetto, E. Biglieri, and R. Daffara, "Modeling and performance evaluation of nonlinear satellite links – a Volterra series approach," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 15, no. 4, pp. 494–507, Jul. 1979.
- [8] M. S. Muha, C. J. Clark, A. Moulthrop, and C. P. Silva, "Validation of power amplifier nonlinear block models," in *IEEE MTT-S Int. Microwave Symp. Dig.*, Anaheim, Jun. 1999, pp. 759–762.