Simplified Volterra Series for the Behavioral Modeling of Dual-band Power Amplifiers under Carriers with Integer Multiple Frequencies

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***Abstract*—In this article, the simplification of the Volterra series is applied to different multiplicities between the carrier frequencies of a dual-band power amplifier. In here, the theory used at previous works is extended to different cases. In all cases, new specific terms are included to maintain the model accuracy. The terms of every different case are identified. Matlab simulations validate the models. A generic model containing all extra terms is created and tested. The results are presented to validate the study.**

Keywords—Behavioral modeling; Power amplifier; Volterra series.

I. INTRODUCTION

The power amplifier has a crucial importance at energy consumption in wireless communication systems. The dual-band power amplifier (DBPA), as an extension of the single-band power amplifier, has the same importance. To obtain low consumption levels, the DBPA must operate at a high power region. Although, in this region, the DBPA presents nonlinearities which interfere the component performance, causing noise and distortions. The operation at the linear region is impracticable for the energy consumption is elevated. An actual solution is a technique called digital predistortion, or DPD, which, when put together with a power amplifier, makes the combination behaves like a linear system.

One of the biggest limitations of the DPD technique is that, for its project to be accomplished, a cheap and easy-to-use computational model of power amplifier is required.

Created by the Italian Vito Volterra, the Volterra series may be used to model a DBPA, because this series can model nonlinear systems with memory [1].

One of the biggest issues in Volterra series is that it generates too many coefficients, becoming difficult its application. Even though, in the previous work of [2], it is proven that, for single-band power amplifiers, only a small number of coefficients is really important for a precise model. The previous research of [3] confirms that the reduction strategy can be expanded to the DBPA case, without generality loss. There is also proof that, for carrier frequencies that are integer multiples of each other, new specific terms must be added to maintain the modeling accuracy.

The contributions of this work are twofold. First, in here, the cases where the carrier frequencies are multiples 3, 4 and 5 times between each other are analyzed. There are also the cases where the carrier frequencies have no integer multiplicity and are one the double of the other, but, these two cases were studied before, and are being used here just for comparison. Second, a generic model that includes the additional terms for each analyzed integer multiplicity (2, 3, 4 and 5) is presented. The accuracy of the different models is assessed based on a case study.

The organization of this work is given as follows. Section II explains the theory used to develop this work. Section III presents the results to validate all the studied cases. The conclusions are shown at Section IV.

II. VOLTERRA SERIES

Dating from 1887, the Volterra series was created by the Italian Vito Volterra, and its capability of modeling nonlinear systems with memory made it known at the world war II, being used to analyze the effect of radar noise in a nonlinear receiver circuit. The Volterra series has adjustable parameters called kernels. A four input Volterra series can be generically described by

$$
\begin{aligned}
y_a(n) = & \sum_{P=1}^{P_0} \sum_{P_1=0}^{P} \cdots \sum_{P_4=0}^{P} \sum_{q_1=0}^{M} \cdots \sum_{q_{P_1}=q_{P_1-1}}^{M} \sum_{q_{P_1+1}=0}^{M} \cdots \\
& \sum_{q_{P_1+P_2}=q_{P_1+P_2-1}}^{M} \sum_{q_{P_1+P_2+1}=0}^{M} \cdots \sum_{q_{P_1+P_2+P_3}=q_{P_1+P_2+P_3-1}}^{M} \sum_{q_{P_1+P_2+P_3+1}=0}^{M} \cdots \\
& \sum_{q_P=q_{P-1}}^{M} h_{\substack{P_1,P_2,P_3,P_4 \\ q_1,q_2,\ldots,q_P}} \prod_{j_1=1}^{P_1} x_1\left(n-q_{j_1}\right) \\
& \prod_{j_2=P_1+1}^{P_1+P_2} x_1^*\left(n-q_{j_2}\right) \prod_{j_3=P_1+P_2+1}^{P_1+P_2+P_3} x_2\left(n-q_{j_3}\right) \\
& \prod_{j_4=P_1+P_2+P_3+1}^{P_1+P_2+P_3+P_4} x_2^*\left(n-q_{j_4}\right)
\end{aligned} \tag{1}
$$

where M is the memory length and P_0 is the polynomial order truncation. The four inputs are represented by x_1, x_2, and their complex conjugates x_1* and x_2*. In (1), the symmetric coefficients were excluded, for it don't cause any influence at the results. Though, even with a more compact model, the number of coefficients generated by (1) grows exponentially with the values of P_0 and M. The subscript "a" refers to the 1*st* or 2*nd* bands.

Another Italian, Sergio Benedetto, in the previous work of [2], showed that for a single-band power amplifier, only a small number of coefficients is important to describe how the power amplifier behaves around the carrier. In the previous work of [3], it's shown that this strategy can be expanded to a DBPA. The others coefficients only contribute at the harmonic frequencies of the carrier. Applying the proposed changes, (1) can be reduced to

$$y_1(n) = \sum_{p=1}^{P} \sum_{P_1=1}^{p} \sum_{q_1=0}^{M} \cdots \sum_{q_{P_1}=q_{P_1-1}}^{M} \sum_{q_{P_1+1}=0}^{M} \cdots \sum_{q_{2P_1-1}=q_{2P_1-2}}^{M} \sum_{q_{2P_1}=0}^{0} \cdots \sum_{q_{P_1+p-1}=q_{P_1+p-2}}^{M} \sum_{q_{P_1+p}=0}^{M} \cdots \sum_{q_{2p-1}=q_{2p-2}}^{M} h_{p,P_1,q_1,q_2,\ldots,q_{2p-1}} \prod_{j_1=1}^{P_1} x_1\left(n-q_{j_1}\right) \prod_{j_2=P_1+1}^{2P_1-1} x_1^*\left(n-q_{j_2}\right) \prod_{j_3=2P_1}^{p+P_1-1} x_2\left(n-q_{j_3}\right) \prod_{j_4=p+P_1}^{2p-1} x_2^*\left(n-q_{j_4}\right) \quad (2)$$

where $P_0 = 2P-1$. To model y_2, the roles of x_1 and x_2 must be changed in (2).

Equation (2) drastically reduces the number of coefficients. Still, the model loses accuracy when frequencies multiple between each other are applied.

Indeed, when the carrier frequencies have an integer multiplicity between each other, new terms must be added, depending on the multiplicity itself. In the work of [3] it was validated for the case where the frequencies were one the double of the other, and the Volterra series were truncated at 5*th* order, with memory length equal to 1.

In this work, the focus is to extend the previous work of [3], now analyzing the cases where the carrier frequencies have integer multiplicities of 3, 4 and 5 times and creating a generic script where all the extra terms are added together, looking for the possibility to input any multiple carriers and still attain good results. Also, the Volterra series is now truncated at 7*th* order, with memory length equal to 1.

III. VALIDATION

The Volterra series from (1) and (2) are now analyzed for P_0=7 and M=1, where (1) creates 6434 coefficients and (2) creates 224 coefficients to estimate each output (y_1 or y_2). In this work, Matlab software scripts were made based on the Volterra series (1) and (2). Moreover, specific scripts for each carrier integer multiplicity were made.

The programs are applied to fit input-output data obtained from a DBPA represented by a Wiener architecture, which utilizes a finite impulse response (FIR) filter with two coefficients and a seventh order polynomial. The complex-valued envelopes that modulate the carriers f_1 and f_2 are, respectively, a 3GPP WCDMA signal with a bandwidth of 8.84 MHz and an LTE OFDMA signal with a bandwidth of 10 MHz. The complex-valued envelopes were captured at a sampling frequency of 61.44 MHz. A total of 5000 samples were collected. The least squares algorithm used 2500 samples to extract the Volterra kernels, leaving the other 2500 samples to the modeling validation. Error signals defined by the difference between desired and estimated output signals were calculated. The normalized mean square error (NMSE), according to [4], was then evaluated.

A. *Case 1: f_1 and f_2 have no integer multiplicity*

In this case, f_1 = 614.4 MHz and f_2 = 2000 MHz. Table I shows the NMSE results obtained using (1) and (2).

TABLE I. NMSE RESULTS IN dB FOR CASE 1

Band	Model	Extraction	Validation
f_1	(1)	-232.9565	21.7107
f_1	(2)	-59. 9024	-48.8486
f_2	(1)	-238.5808	16.8703
f_2	(2)	-61.9960	-48.9472

Table I shows that the error obtained from the extraction data, using (1), is very low. That occurs due to the high quantity of coefficients, creating a perfect model for the extraction data. The validation error is too high for the same reason. The coefficients acquired from extraction are set perfectly for the extraction data. Any different data applied to this identified model will create points in completely different places. Equation (2) creates a much smaller group of coefficients, and due to this, the error for the extraction data is higher than for (1). However, the model created can be used with any package of data, and that is proved by the validation NMSE.

B. *Case 2: $2f_1=f_2$*

In this case, f_1 = 614.4 MHz and f_2 = 1228.8 MHz. Table II shows the NMSE results obtained using (2) and its modified version, called (2) mod., that includes the additional terms reported in Table III.

TABLE II. NMSE RESULTS IN dB FOR CASE 2

Band	Model	Extraction	Validation
f_1	(2)	-24.6386	-15.7751
f_1	(2) mod.	-63.3634	-48.3028
f_2	(2)	-27.3703	-16.6879
f_2	(2) mod.	-66.2316	-48.5571

Because f_2 is an integer multiple of f_1, the additional terms reported in Table III must be included in (2).

TABLE III. NEW TERMS FOR CASE 2

Band	New terms	Total coefficients
f_1	(X2.X1*) (X2.X1*)(X1.X1*) (X2.X1*)(X2.X2*) X1(X2*.X1.X1) (X2.X1*)(X2.X1*.X1*) X1(X1.X1*)(X2.X1*.X1*) X1(X1.X1*)(X2*.X1.X1) X1(X2.X2*)(X2.X1*.X1*) X1(X2.X2*)(X2*.X1.X1) (X2.X1*)(X2.X2*)(X2.X2*) X1(X2.X1*.X1*)(X2.X1*.X1*) X1(X2*.X1.X1)(X2*.X1.X1) (X2.X1*)(X2.X1*.X1*)(X2.X2*)	480 (13 new terms)
f_2	(X1.X1) (X1.X1)(X1.X1*) (X1.X1)(X2.X2*) X2(X2.X1*.X1*) (X1.X1)(X2*.X1.X1) X2(X1.X1*)(X2.X1*.X1*) X2(X1.X1*)(X2*.X1.X1) X2(X2.X2*)(X2.X1*.X1*) X2(X2.X2*)(X2*.X1.X1) (X1.X1)(X1.X1*)(X1.X1*) X2(X2.X1*.X1*)(X2.X1*.X1*) X2(X2*.X1.X1)(X2*.X1.X1) (X1.X1)(X1.X1*)(X2*.X1.X1.)	462 (13 new terms)

Equation (2) presents a bad performance for the fact that the carrier frequencies are multiple between each other. When extra terms are added to (2), creating (2) modified, the performance is significantly improved.

C. *Case 3: $3f_1=f_2$*

In this case, f_1 = 614.4 MHz and f_2 = 1843.2 MHz. Because f_2 is an integer multiple of f_1, the additional terms reported in Table IV must be included in (2).

TABLE IV. NEW TERMS FOR CASE 3

Band	New terms	Total coefficients
f_1	(X2.X1*.X1*) X1(X2.X1*.X1*.X1*) X1(X2*.X1.X1.X1) (X2.X1*.X1*)(X2.X2*) (X2.X1*.X1*)(X2.X1*.X1*.X1*) (X2.X1*.X1*)(X1.X1*)(X1.X1*) (X2.X1*.X1*)(X1.X1*)(X2.X2*) (X2.X1*.X1*)(X2.X2*)(X2.X2*) X1(X1.X1*)(X2*.X1.X1.X1) X1(X2.X2*)(X2*.X1.X1.X1)	460 (10 new terms)
f_2	(X1.X1.X1) (X1.X1.X1)(X1.X1*) (X1.X1.X1)(X2.X2*) X2(X2.X1*.X1*.X1*) X2(X1.X1*)(X2.X1*.X1*.X1*) X2(X1.X1*)(X2*.X1.X1.X1) X2(X2.X2*)(X2.X1*.X1*.X1*) X2(X2.X2*)(X2*.X1.X1.X1) (X1.X1.X1)(X2*.X1.X1.X1) (X1.X1.X1)(X1.X1*)(X1.X1*)	436 (10 new terms)

Table V shows the NMSE results obtained using (2) and its modified version, called (2) mod., that includes the additional terms reported in Table IV.

TABLE V. NMSE RESULTS IN dB FOR CASE 3

Band	Model	Extraction	Validation
f_1	(2)	-27.0774	-16.7229
f_1	(2) mod.	-61.0868	-46.3332
f_2	(2)	-32.2328	-22.0990
f_2	(2) mod.	-64.5057	-47.4720

Just as before, the error in (2) is bigger than the maximum accepted. Though, with the new terms added, the NMSE results shown are far better.

D. *Case 4: $4f_1=f_2$*

In this case, f_1 = 614.4 MHz and f_2 = 2457.6 MHz. Because f_2 is an integer multiple of f_1, the additional terms reported in Table VI must be included in (2).

TABLE VI. NEW TERMS FOR CASE 4

Band	New terms	Total coefficients
f_1	(X2.X1*.X1*.X1*) X1(X2.X1*.X1*.X1*.X1*) X1(X2*.X1.X1.X1.X1) (X2.X1*.X1*.X1*)(X2.X2*)	288 (4 new terms)
f_2	(X1.X1.X1.X1) X2(X2.X1*.X1*.X1*.X1*) X2(X2*.X1.X1.X1.X1) (X1.X1.X1.X1)(X1.X1*)	276 (4 new terms)

Table VII shows the NMSE results obtained using (2) and its modified version, called (2) mod., that includes the additional terms reported in Table VI.

TABLE VII. NMSE RESULTS IN dB FOR CASE 4

Band	Model	Extraction	Validation
f_1	(2)	-40.8177	-32.5696
f_1	(2) mod.	-60.6161	-48.5335
f_2	(2)	-47.1141	-37.2589
f_2	(2) mod.	-62.5285	-48.3406

The results are the same as seen before. The modified model improves the NMSE results by adding few new terms.

E. *Case 5: $5f_1=f_2$*

In this case, f_1 = 614.4 MHz and f_2 = 3072 MHz. Because f_2 is an integer multiple of f_1, the additional terms reported in Table VIII must be included in (2).

TABLE VIII. NEW TERMS FOR CASE 5

Band	New terms	Total coefficients
f_1	(X2.X1*.X1*.X1*.X1*) X1(X2.X1*.X1*.X1*.X1*.X1*) X1(X2*.X1.X1.X1.X1.X1) (X2.X1*.X1*.X1*.X1*)(X2.X2*)	302 (4 new terms)
f_2	(X1.X1.X1.X1.X1) X2(X2.X1*.X1*.X1*.X1*.X1*) X2(X2*.X1.X1.X1.X1.X1) (X1.X1.X1.X1.X1)(X1.X1*)	286 (4 new terms)

Table IX shows the NMSE results obtained using (2) and its modified version, called (2) mod., that includes the additional terms reported in Table VIII.

TABLE IX. NMSE RESULTS IN DB FOR CASE 5

Band	Model	Extraction	Validation
f_1	(2)	-48.5174	-39.9026
f_1	(2) mod.	-60.6999	-48.4478
f_2	(2)	-52.5354	-41.0897
f_2	(2) mod.	-61.9209	-47.0777

Same as always, the new terms on the modified model can improve the accuracy of the DBPA.

It can be noticed that accordingly to the growth of the multiplicity, the new terms begin to appear only in the bigger orders, and decrease their quantity. For example, when the multiplicity is 5, like in case 5, the first new terms only appear in the 5^{th} order. It occurs to all the integer multiplicities, that's why case 2 has 13 new terms and case 5 has only 4. Also, usually, not generally, the more new terms you have the more coefficients. That can't be a rule because it depends specifically on the terms added. There are terms that create more coefficients than others, due to the combination of inputs.

F. Case 6: generic model

In this case, all data packages were used and all NMSE collected. The generic model has all extra terms from all the cases analyzed in this work, totalizing 31 new terms for f_1 (858 coefficients) and also 31 for f_2 (788 coefficients). The first expectancy was that some terms from different cases should be equal. This way, the generic model would utilize just one of them and the results would be better. However, the only appearance of equal terms was from different bands, e.g. the term X2(X1.X1*)(X2.X1*.X1*.X1*) which is found in case 3, in band f_2, and the term X1(X2.X1*.X1*)(X2.X1*.X1*) which is found in case 2, band f_1, are equal. The problem is that when the equal terms are found in different bands, the generic model can't take advantage from it.

Table X shows the NMSE results obtained using the generic model for all the carriers with integer multiplicities studied in this work and the case where the carriers have no integer multiplicity.

TABLE X. NMSE RESULTS IN DB FOR CASE 6

Data applied	Band			
	f_1		f_2	
	Extraction	*Validation*	*Extraction*	*Validation*
f_1=614.4 MHz f_2=2000 MHz	-62.7457	-37.6648	-64.6536	-42.7010
f_1=614.4 MHz f_2=1228.8 MHz	-65.7117	-39.4559	-68.4471	-43.2871
f_1=614.4 MHz f_2=1843.2 MHz	-62.9589	-40.1624	-67.5297	-41.3023
f_1=614.4 MHz f_2=2457.6 MHz	-63.5355	-39.4087	-65.6576	-40.7489
f_1=614.4 MHz f_2=3072 MHz	-63.6397	-39.7308	-65.0794	-39.9906

Figure 1 shows the output amplitude as function of the input amplitude for y_1 modeling in case 2. Measured data represents the output of the DBPA. Simulated 1 represents the Volterra series of (2), without adding the extra terms. Simulated 2 represents the output of the generic model, where all the extra terms for all cases were added. Equation (2) modified for case 2 was not included in the figure because the results were very close to Simulated 2.

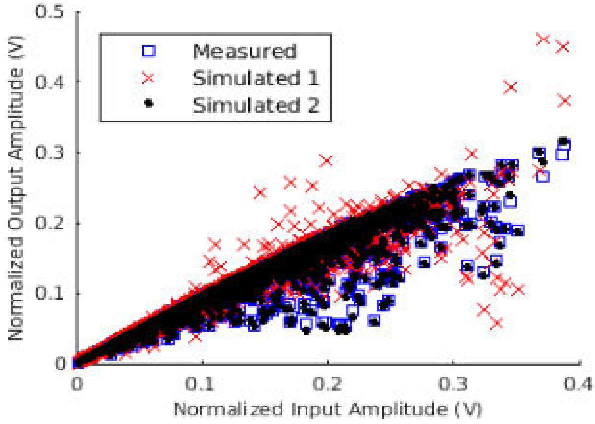

Fig. 1. Output amplitude as function of input amplitude for y_1 modeling in case 2, where Measured represents the output of the DBPA, Simulated 1 represents the Volterra series of (2) output and Simulated 2 represents the generic model output.

It can be seen that Simulated 1 presents an error much higher than Simulated 2, as seen in Tables II and X.

IV. CONCLUSION

As shown in previous works, to model DBPAs using Volterra series, a great amount of attention should be paid in the addition of the extra terms, because every case demands specific terms, and that should be calmly analyzed. The construction of a generic model for DBPAs using Volterra series can present satisfactory results as long as all the terms are correctly added to the model. The addition of too many terms causes the generic model to be more inaccurate than the specific models, but, still can be used for general purposes.

AKNOWLEDGEMENT

The authors would like to acknowledge the financial support provided by Programa de Iniciação Científica da Universidade Federal do Paraná, modalidade PIBIC UFPR/Tesouro Nacional, and by Pró-Reitoria de Assuntos Estudantis (PRAE-UFPR).

REFERENCES

[1] V. Mathews and G. Sicuranza, Polynomial Signal Processing. New York: Wiley, 2000.

[2] S. Benedetto, E. Biglieri, and R. Daffara, “Modeling and performance evaluation of nonlinear satellite links – a Volterra series approach,” IEEE Trans. Aerosp. Electron. Syst., vol. 15, no. 4, pp. 494–507, Jul. 1979.

[3] T. M. Dompsin, O. A. P. Riba, and E. G. Lima, “Behavioral Modeling of Dual-band Radio Frequency Power Amplifiers using Volterra Series,” in XV Microelectronics Students Forum, Salvador, Sep. 2015, pp. 1–4.

[4] M. S. Muha, C. J. Clark, A. Moulthrop, and C. P. Silva, “Validation of power amplifier nonlinear block models,” in IEEE MTT-S Int. Microwave Symp. Dig., Anaheim, Jun. 1999, pp. 759–762.