A Study of the Geometrical Correction Factor and the Membrane Thickness on the Sensitivity of the Transversal Piezoresistive Pressure Sensor

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ABSTRACT

The sensitivity of four-terminal devices like Hall and piezoresistive sensors is very dependent on its geometric parameters. This dependence is modeled by the Geometrical Correction Factor (*G*). The majority of these studies take very time-consumption analytical calculations for the analysis of *G*. In order to simplify this analysis, we present numerical analyses using FEM (Finite Element Method) for the most common geometrical forms of four-terminal devices. This result is general for any four-terminal-shaped sensors and can be used to optimize the sensor aspect ratio leading the maximization of *G*. In addition, FEM was also used to evaluate the membrane's thickness influence over the sensor sensitivity by analyzing the in-plane mechanical stress behavior on the sensor active area. Experimental result of a new topology of pressure sensor is also presented, which maximizes *G* in comparison with conventional four-terminal devices and also improves its sensitivity. A special designed anisotropic wet etching system was used to post-process the sensor membrane. This anisotropic wet etching system allows the fabrication of well-defined membranes with thickness of 20 μ m ± 3 μ m and roughness as low as 90 nm rms.

Index Terms: piezoresistive sensors, piezoresistance effect, short-circuit effect, high sensitivity and CMOS microsystem.

1. INTRODUCTION

Since the first four-terminal device came out as a sensing element for either pressure or Hall sensors, many studies have been performed toward the minimization of the dependence of these four-terminalshaped devices on its geometric parameters and the maximization of its sensitivity [1-4]. This negative dependence is due to the finite size of the current and potential terminals. This dependence, normally called short-circuit effect, is modeled by the Geometric Correction Factor (G) and should be taken into consideration during design of the devices.

The calculation of G can be performed both analytically and numerically. Even though analytical methods provide a suitable approach, they are usually performed using mathematical technique of conformal transformations, which requires arduous and very time-consumption calculations [1-4]. Thus, a numerical method to calculate G becomes a very attractive approach. Furthermore, it does not require the designer to make simplifying assumptions concerning the boundary conditions and it does not neglect parts of the mechanical stress tensors.

In this paper we investigated G for various moststudied geometries of the CSR (Current Spread Region) of the devices through the numerical analysis using Finite Element Method (FEM). For that, we consider silicon pressure sensors with a shaped-rectangular active area (CSR) of p-type silicon. By considering that the majority of the current flow is confined on the surface, a two-dimensional mathematical model can be used for calculations of sensor's output voltage and, hence, for FEM analysis of G. For a more accurate result, the CSR's thickness or depth should should be as small as possible in order to achieve a minimum of current flowing toward the substrate. The study developed throughout this paper is valid for both piezoresistive and Hall Effect devices since the mechanical stress has a similar effect to the anisotropic resistivity of silicon, as the magnetic field in Hall effect [5]. Hence, the piezo-Hall effect is closely related to the piezoresistive effect [6].

The silicon membrane is a mechanical conveyor. The differential pressure applied to opposite sides of the membrane is converted in an in-plane mechanical stress on the membrane surface. Thus, the pressure sensors sensitivity also relies on the behavior of the in-plane mechanical stresses that acts over their active region. The amount of mechanical stress is dependent on the membrane geometry (thickness, width and length) as well as its roughness and the orientation of the crystallographic axes. The membrane thickness and roughness varies according to its microfabrication process. FEM analysis was evaluated to determine the relation between membrane thickness and the in-plane stress over the active area of the piezoresistive sensor. In order to minimize the influence of these factors we used a special designed anisotropic wet etching to post-process the membrane of the pressure sensor.

2. AN ANALYTICAL APPROACH OF THE PRES-SURE SENSOR AND THE STRESS IN A SQUARE MEMBRANE

A. Study of the Output Voltage of the Pressure Sensor

Figure 1 presents both a FTPS and a new topology of pressure sensor, which was presented in a former work [7]. This new sensor is referred to as Multi-Terminal Pressure Sensor (MTPS) and is based on the Transversal Piezoresistive Effect. This sensor is a particular case of the Four-Terminal Pressure Sensors (FTPSs), which is formed by merging four FTPSs together.



Figure1. Geometric parameters of a MTPS merged from four FTPS.

In Figure 1, L and W are the length and the width of the device, respectively. l and w are the length and the width of the current terminals, respectively. S is the width of the sensing-contacts. Terminals 1's and 2's represent the current terminals and 3 and 4 the voltage-sensing contacts. *Gap* is the distance between two current terminals for the MTPS.

Once FTPS's output voltage is known (8), the calculation for the output voltage of a MTPS is straightforward since it works like four FTPSs. Hence, the sensitivity of the MTPS is increased proportionally in accordance with the number of input current terminals. Considering the sensors are biased by a constant current source I_{bias} , MTPS's output voltage is written as [8]:

$$V_{MTPS} = \frac{n}{2} V_{FTPS},$$
(1)
where $V_{FTPS} = \frac{1}{2} \pi_{44} \left(\sigma_1 - \sigma_2 \right) \rho_s I_{bias} G,$

where *n* is the number of the input current terminals, π_{44} is the shear piezoresistive coefficient for ptype silicon, ($\sigma_1 - \sigma_2$) is the resultant uniaxial mechanical stress along the <110> crystallographic orientations, ρ_s is the sheet resistance of a nonstrained silicon.

Note that G is included in equation (1) and is always G < 1 [6]. Thus, G models the decay of the output voltage by the short-circuit effects due to current and sensing terminals. As these terminals are made of metal electrodes, they provide a low-resistance path for the current density lines and a lowresistance region for equipotential lines.

B. Study of the Behavior of the Stress in a Square Membrane

Silicon pressure sensors are usually fabricated over a bulk-micromachined membrane. The sensor sensitivity is rather dependent on the membrane properties such as surface roughness, orientation, alignment and geometry [9]. Such properties determine how in-plane stress will act over the active area of the sensor.

The in-plane stress in an anisotropic wet etched silicon membrane can be approximated by a square membrane with all edges clamped. A square membrane under a differential pressure p deflects and a stress appears over its surface. The maximum in-plane stress as a deflection result is given by [10]:

$$\sigma_{\max} = \Psi p \frac{a^2}{h^2} \tag{2}$$

where ψ is a constant related to the membrane mechanical properties and geometry, *a* is the membrane width and *b* the membrane thickness. For a square membrane with all edges clamped, ψ values are approximately 0.3 [10]. Maximum stress occurs at the center of the membrane edges. Despite of silicon has anisotropy mechanical properties, isotropy is assumed. This assumption is acceptable for stress estimative.

Equation (2) allows an estimative of the membrane maximum stress, but for a more precise stress evaluation, analytical methods became too complex. In this case, FEM analysis provides an accurate result for the maximum in-plane stress and also depicts the stress distribution in the membrane, as the different stress components and the maximum stress location.

3. AN NUMERICAL APPROACH OF THE GEOMETRICAL CORRECTION FACTOR AND THE DEPENDENCE OF THE SENSITIVITY OVER VARIATION OF THICKNESS OF THE MEMBRANE

A. Study of the In-Plane Mechanical Stress over Variation of the Thickness of a Square Membrane

Numerical simulations using FEM were performed in order to predict the magnitude of the inplane stress by varying the membrane's dimensions and, hence, analyze its influence on the pressure sensors' sensitivities. A 3D model based on a real membrane topology was created using finite element AnsysTM software, as shown in Figure 2.



Figure 2. Numerical 3D model of a square membrane.

Figure 3 shows the simulation result for stress along x direction in a membrane of 2 mm x 2 mm x $20 \mu m$.

Figure 4 shows the maximum stress versus membrane thickness while its width was fixed at 2 mm and its thickness varied from 20 μ m to 100 μ m. A small difference can be observed between analytical and numerical analysis related to the simplifications assumed in the first one. As analytical analysis predicts an in-plane stress proportional to the thickness to the power of -2, the numerical analysis predicts an in-plane stress proportional to thickness to the power of 2.16.



Figure 3. In-plane mechanical stress behavior over a square membrane along \boldsymbol{x} direction.



Figure 4. In-plane mechanical stress over a variation of the membrane thickness.

Note that the sensitivity change due to a positive variation on thickness is different from a negative variation. For a variation of r in the membrane thickness, a variation in the sensor sensitivity is given by:

$$S = S_0 (1 \pm r)^{-2}$$
(3)

where S_0 is the sensitivity for a specific membrane thickness. The membrane thickness control is therefore critical for the sensor sensitivity repeatability.



Figure 5. Resultant maximum in-plane mechanical stress over a square membrane along x direction.

As predicted by equation (1), the output voltage of the sensors is a function of two stress components: $\sigma_{xx} - \sigma_{yy}$. Based on FEM, this resultant maximum in-plane mechanical stress amounts to 100 MPa. Figure 5 shows the magnitude of both: σ_{xx} and σ_{yy} , as well as the resultant $\sigma_{xx} - \sigma_{yy}$ along the membrane length.

B. Study of the Geometrical Correction Factor over Aspect Ratio Variation of the Pressure Sensor

As aforementioned, the output voltage of likefour-terminal pressure sensors is strongly dependent of their ohmic terminals. Figure 6 depicted the shortcircuit effect on a four-terminal device.



Figure 6. Short-Circuit Effect: current density lines (upper) and equipotential lines (lower) in the left-hand side and numerical analysis in the right-hand side.

FEM is well suitable to the solution of the differential equations with well-known boundary conditions. FEM allows the analysis of complex geometries by subdividing them into a finite number of more simply shape elements through a process called meshing [11]. Theoretically, the more numbers of elements the more accurate is the result of the simulation. All simulations were performed using AnsysTM 8.1.

The geometries chosen for analysis of G are shown in Figure 7. As an example, Figure 7 also shows a meshed geometry of a MTPS with boundary conditions applied to it.



Figure 7. Geometric parameters (left-hand side) of various geometries and meshed structure of a MTPS (right-hand side).

The design of a pressure sensor requires both: a structural and an electrical analysis. Thus, we performed a multi-physics analysis using two types of finite elements: PLANE183 (structural) and PLANE223 (electrical). Still, as the majority current flow can be assumed to be confined on the CSR surface, all models of geometries used in the simulations are 2D-shaped. The sensors are designed on a square membrane and rotated by an angle of 45 degrees in relation to the [110] direction for maximum sensitivity [8]. The whole membrane region was assumed to be under a uniform stress of 100 MPa. The CSR is a layer of p-type silicon with a resistivity of 78 k Ω µm. The sensor was biased by a constant current of 2 µA. G was obtained by comparing the numerical output voltage against the analytical one.

FTPS Without Sensing Terminals

In this analysis we varied S/L ratio from about 0.01 to 0.5 while kept W/L varying from 0.2 to 1. W was of 20 µm. Figure 8 shows our results compared against analytical results of Gridchin *et al* [12].

Journal Integrated Circuits and Systems 2010; v.5 / n.2:140-147



Figure 8. A comparison between numerical and analytical results for *G* of a FTPS without sensing terminals.

Output voltage amounted to approximately 10.7 mV when G is close to 1. This value of output voltage remains the same for any four-terminal pressure sensor as its G is ideal.

FTPS With Sensing Terminals

Before analyzing the influence of L/W, we first analyzed the influence of the sensing contacts when terminals are added to it. Our analysis shows that for a ratio of $l/S \ge 1$ the influence of the sensing contacts can be neglected. Then, we kept S/W = 0.15 and varied L/W from 0.1 to 4. W was of 20 µm. Figure 9 shows our results compared against experimental results of Bao *et al* [13]. Note that even though we are not considering punctual sensing-contacts, G approaches to 1 like an ideal FTPS (with very small sensing-contacts and large L/W).

Greek Cross

Greek Cross is a symmetrical geometry where L is a function of h. Thus, we varied h so we modified L. As a result, L/W was varied from 1.03 to 7 in steps of 0.25 and h/k varied from 0.015 to 3 in random steps. W was of 20 µm. Figure 10 shows our results compared against analytical results of Gridchin *et al* [14].



Figure 9. A comparison between numerical and experimental results for *G* of a FTPS with sensing terminals.

A Study of the Geometrical Correction Factor and the Membrane Thickness on the Sensitivity of the Transversal Piezoresistive Pressure Sensor Coraucci. Oliveira. Garcia & Fruett



Figure 10. Comparison between numerical and analytical results for *G* of a Greek Cross.

FTPS Modified

The geometry of the FTPS Modified is based on the FTPSs although its geometric parameters are referred to differently. This is needed so we can extrapolate this device to a multi-terminal one, as shown in Figure 1. For this analysis we varied l/wfrom 0.01 to 2 in random steps while kept L/W varying from 0.01 to 3. S/W was kept as small as possible and ranged from 0.01 to 0.02. W was of 60 µm. Figure 11 shows the numerical results.



Figure 11. Numerical analysis of G for a FTPS Modified.

MTPS

The analysis of the MTPS was performed as for the FTPS Modified. Here again, we varied l/w from 0.01 to 2 while kept L/W varying from 0.01 to 3. S/W was kept as small as possible (ranging from 0.01 to 0.03). W was of 180 µm. Figure 12 shows the numerical results.

Output voltage amounted to approximately 43.1 mV when G is close to 1. Figure 13 shows a comparison of G between both FTPS Modified and MTPS with l/w = 1.

Optimum Aspect Ratios

Table I presents aspect ratios for G of approximately 1. Recall that due to technology limitation some of these values may be unfeasible and a change



Figure 12. Numerical Analysis of the correction geometrical factor, *G*, of a MTPS.



Figure 13. Correction Factor, *G*, for both FTPS and MTPS with l/w = 1.

is needed for fabrication. However, this change can affect G strongly, as it is the case if S/W of the FTPS without sensing terminals is increased.

TABLE I. Optimum Aspect Ratios for G maximized (≈ 1)				
Topology	L/W	S/W	l/w	l/S
FTPS Without Sensing Terminals	3.75	0.01	-	-
FTPS With Sensing Terminals	3.25	0.15	-	1
FTPS Modified	0.7	0.01	1	-
MTPS	0.1	0.01	1	-

4. EXPERIMENTAL RESULTS

A. Microfabrication of the Piezoresistive Sensing Elements

The sensors were fabricated using a microelectronic process available at the Center for Semiconductors Components (CCS-UNICAMP). The CSR is defined by a p-type implantation over an n-type (100) silicon wafer. The sheet resistance of the active area is about 1.5 k Ω . L/W = 0.5 was chosen for both devices. The sensor fabrication also includes the n-type implantation for guardring and aluminum deposition for metallization. Figure 14 shows the photograph of the MTPS fabricated.

B. Microfabrication of the Membrane

Membranes for silicon pressure sensors are usually fabricated using anisotropic wet etching [15]. The in-



Figure 14. Photograph of the MTPS fabricated.

plane stress in a square membrane varies strongly over a variation of thickness and width of the membrane, as aforementioned. In order to minimize the influence of these factors we used a special designed anisotropic wet etching system. Figure 15 shows the silicon etching rate for temperature ranging from 70 $^{\circ}$ C to 90 $^{\circ}$ C.

Figure 16 shows the average rms roughness of the micromachined square membrane.

This anisotropic wet etching system allows the fabrication of well-defined membranes with thickness of 20 μ m ± 3 μ m and roughness as low as 90 nm rms.

After the characterization of our anisotropic wet etching system, we fabricated a membrane as part of the piezoresistive pressure sensor.

The square membrane obtained by postprocess has $1800 \mu m$ of width and $20 \mu m$ of thickness. Figure 17 shows a photograph of the piezoelements in a cleaved membrane.



Figure 15. Anisotropic wet etching rates over a variation of temperature.



Figure16. Average roughness of a square membrane using anisotropic etching in KOH.

Journal Integrated Circuits and Systems 2010; v.5 / n.2:140-147



Figure 17. Photograph of the Four and Multi terminals pressure sensors on a cleaved square membrane.

C. Conditioning Circuit

The conditioning circuit shown in Figure 18 is used to bias the MTPS. For $I_{bias} = 100 \ \mu$ A, a bias voltage of ±3 volts and $R_{bias} = 9 \ k\Omega$ were needed. Note that for the FTPSs only one part of the circuit, which is enough to bias one current terminal, is used.



Figure18. Conditioning Circuit for FTPSs and MTPS.

D. Transfer Functions

Figure 19 shows the experimental results for both FTPS and MTPS under the bias condition aforementioned. For sake of simplifying the comparison between the results, sensors' offsets were omitted in Figure 19.

The sensitivity for both FTPS and MTPS amounts to 1.55 mV/psi and 4.80 mV/psi, respectively.



Figure19. Experimental result of the sensitivity of both FTPS Modified and MTPS.

5. DISCUSSION

Our study shows that the mechanical stress in bulk-micromachined membranes is strongly dependent of its aspect ratio. The results presented in Figure 4 show that the maximum stress is a function of the membrane thickness. A 10% thickness variation will lead up to a 23% variation on maximum stress. This maximum stress is found to be inversely proportional to membrane thickness and is located nearby the center of the membrane edges.

Other factor that effectively affects the sensor sensitivity is the short-circuit effect modeled by *G*. The results presented in Figure 8 shows that the sensitivity of four-terminal devices without sensing terminals is strongly dependent on the sensing-contacts while not punctual. Thus, a sensor with this type of geometry should be designed with *L* enough long due to technology limitation for designing of the sensing-contacts. Now, it is clear from Figure 9 that when terminals are added to the sensing-contacts the influence of *G* is minimized for $l/S \ge 1$. Under this condition, the sensing contacts behave as punctual ones.

The Greek Cross also presents an improved G factor for large L/W. Taking an analogy between l/S of a FTPS and h/k of the Greek Cross, we realize that they are the same parameters and, hence, it is clear that for $L/W \ge 3G$ is maximized, as shown in Figure 10. Still, due to symmetry of this geometry, offset cancellation techniques can be applied to it [16].

A careful attention should be taken into account for comparing the numerical results of either a FTPS Modified or a MTPS against the ones of conventional FTPSs. Recall that the geometric parameters were modified and, therefore, a direct comparison between both results should not be suitable. Stead, we compared the results of a FTPS Modified against the one of MTPS, as show in Figure 13. From Figure 13, our results show that *G* is improved for the MTPS. As *W* of the MTPS is much greater than the one of the FTPS Modified it is expected that L/W is much less than the one of FTPS Modified for the same *G*. However, even if we normalize *W* of both devices we would notice an improvement for *G* of the MTPS.

In a general view, for any these devices, G is maximized when L/W is enough large. Comparisons between our results and the other researchers show that numerical analysis using FEM has a suitable approach for analyzing of G.

Numerical and experimental results show that the sensitivity of the MTPS is a function of the number of input current terminals. Numerical results reported an output voltage for the MTPS of about 43 mV and about 11 mV for the four-terminal devices. Experimental results reported sensitivity of about 1.5 mV/psi for the four-terminal devices and of about 4.8 mV/psi for the MTPS, as shown in Figure 19. Our results show that there is flexibility for designing of four and multi terminals sensors regarding the G factor. Aspect ratio of the geometric parameters should be chosen concerning the input and output resistances. Though we can improve G by modifying the aspect ratio, those parameters are also modified accordingly what impacts on the increase of output noise and power consumption.

It is also important to note that the post-process micromachining repeatability is essential to minimize the sensor sensitivity variation and also to avoid over-etching of the membrane.

Figure 16 shows that the roughness of the membrane can be smaller than 90 nm rms.

6. CONCLUSION

Numerical analysis using Finite Element Method shows itself suitable for analyzing of the correction geometrical factor of four-terminal-shaped devices and for predicting their output voltages as well as for optimizing of the membrane's design. Numerical analyses for most-studied four-terminal devices and for in-plane mechanical stress in a square membrane are presented.

Our results show that PSEs' and membrane's geometric parameters must be chosen properly in order to improve the sensor sensitivity. A new topology of multi-terminal piezoresistive pressure sensor (MTPS) is presented. Numerical results show that the geometrical correction factor of the MTPS is improved in relation to the one of conventional devices. Numerical and experimental results show that MTPS's sensitivity is a function of the number of input current terminals. Experimental results for the sensitivities of both FTPS and MTPS amount to 1.55 mV/psi and 4.8 mV/psi, respectively.

Results for the study of the in-plane mechanical stress over a variation of membrane thickness show that the stress is an inverse function of the thickness and very dependable of the membrane's aspect ratio. The thinner is the membrane the stronger is the stress on its surface. As a result, the backside bulk-machining process must be very well controlled in order to realize well-defined membranes. This anisotropic wet etching system allows the fabrication of well-defined membranes with thickness of 20 μ m \pm 3 μ m and roughness as low as 90 nm rms.

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