

# Elastic Coefficient model of a Comb-Drive Hyper Static Beam Validation

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## Abstract

*MEMS are micro-electro-mechanical systems. An example of MEMS is the ‘comb-drive’ structure, whose analysis and modeling have been the motivation for the creation of a Toolbox to simulate it. This paper shows the validation of the mathematic models used for elastic coefficient. The software used as comparison is SUGAR V0.5 that is an open code MatLab program.*

## 1. Introduction

MEMS (micro-electro-mechanical-systems) are microsystems used in the transduction of electromechanical energy, whose functioning is based on the generation of the force through capacitive principle. The structure studied is a MEMS, composed by two "combs" one beside the other, where one is fixed and the other oscillates approaching and moving away itself from the fixed one. This structure can be used as micro-relay, measurer of moistness, accelerometer, etc [LIN98]. The comb-drive modeling is given by its approach to a mass-spring-damp system, where the main difficulties are the calculus of these three basic parameters: effective mass, spring constant and damping coefficient. After these three parameters are calculated, the solution of the equation that represents the behavior of the system is well known. The related one is a differential ordinary equation of second order[ROM98].

With this model and the MATLAB computational tool we expect to create a toolbox that simulates different structure types for general use. We have as objective validate the created toolbox module, trough it's coefficients (effective mass, elastic coefficient and damping coefficient) validation, but unfortunately the software used as comparison (SUGAR) was not able to provide the effective mass and damping coefficient, so we are going to validade only the elastic coefficient in this paper.

## 2. Comb-Drive's Elastic Coefficient Model

In Fig.1 we have a simple model of comb-drive. The fixed part, in solid color, and the mobile part, striped. The functioning of this device is simple to understand: when a potential difference is applied between the contacts (1) and (2), the positive charges will move towards to the fixed "fingers" and the negative charges to the mobile "fingers" (considering the potential in (1) higher than in (2)).

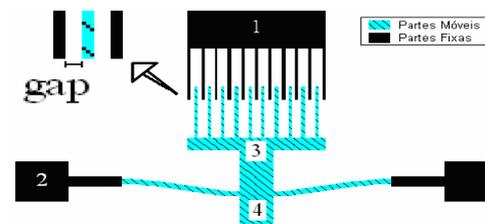


Fig. 1- Comb-drive basic structure.

With the mutual attraction of this charge it appears a resultant force between the two "combs" that can induce the movement of the mobile part. With this knowledge we can model this system as a mass-spring-damp system, whose is well known by mechanics.

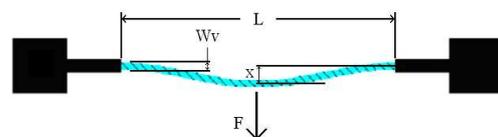


Fig.2 – Beam illustration.

Despite the apparent problem simplicity, the calculation of the parameters M (mobile part effective mass), D (damping) and K (beam elasticity) are not trivial[MAN05]. The K parameter is calculated considering a hyper static beam that works only in elastic regimen. The mobile part effective mass does not enclose all the mobile part, as we can infer in Fig.1, the entire beam does not oscillate on the maximum amplitude. And finally the D parameter depends on the fluid viscosity in which the structure is immersed, being directly proportional to its

viscosity. Let's focus on the K (beam elastic coefficient) parameter.

The K parameter can be calculated by the following equations (see Eq.1 and Eq.2):

$$K = \frac{192 \cdot E_{\text{poli}} \cdot I}{L^3} \dots (Eq.1) \quad I = \frac{e \cdot (W_v)^3}{12} \dots (Eq.2)$$

Where 'E<sub>poli</sub>' is the Young modulus, 'I' is the beam moment and 'L' its length. The 'I' is calculated by the Eq.2, where 'e' is the thickness of the poly silicon layer (whose depend on the fabrication process), and 'W<sub>v</sub>' is the beam width. This is the simplest model found in the literature for an hyperstatic beam. In Eq.3 we have that the electrical force is equal to the elastic constant of the beam multiplied by the maximum displacement of beam (in a DC analysis).

$$F_e = K \cdot x \dots (Eq.3)$$

### 3. Model Analysis and Comparison

To validate the model, two plots were analysed comparing our software to SUGAR V0.5. These plots are: Beam compriment x Displacement, Beam width x Displacement, the K wasn't plotted directly because of SUGAR's limitaions. Manipulating Eq.3 it is obtained Eq.4(we validate our model considering a constant force of 10uN).

$$K = \frac{x}{F} \dots (Eq.3) \quad K = x \cdot 10^5 \dots (Eq.4)$$

So, if we make the force constant for the two simulations, we can compare the displacements. (See Fig.3 and Fig.4) In both graphics the circles plot is our model, the continuous line is SUGAR linear beam model and the striped line is SUGAR non-linear beam model.

As we can see throught the graphics, the model used by us is the same used in the linear model of SUGAR.

### 4. Conclusion

The model used is in agreement with the linear model used on SUGAR. As future works we expect to validade the other parameters and porpouse new models for the parameters of the mass-spring-damp system with more contibutios and more considerations to aproximate it to the reality.

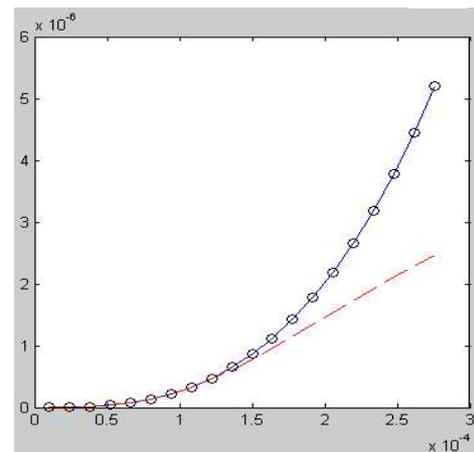


Fig.3 – Beam length x displacement.

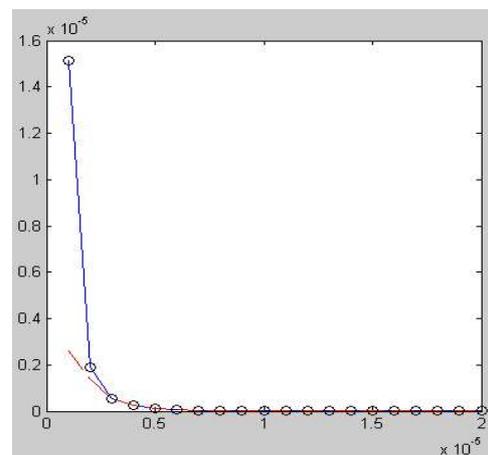


Fig.4 – Beam width x displacement.

### 5. Acknowledgment

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### 6. References

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