Comparison between the depletion approximation and numerical methods results

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A comparison between the results obtained for charge density, eletric potential and eletric field in a p-n junction using the depletion approximation and numerical methods is made in order to analyse and stablish limits for using the depletion approximation with acceptable agreement. It is shown that the depletion approximation is accurate when reverse bias is applied and not accurate at forward or at zero bias condition. Besides, at larger doping concentration the accuracy is increased.

INTRODUCTION

In order to obtain analytical results (specially for charge density, eletric field and potential) along p-n junctions, an approximation has been made to simplify the solution of Poisson's equation[1]. This approximation, called depletion approximation, essentially, says that [2]: all free carriers in excess of the intrinsic concentration have been removed from the depletion region, leaving behind ionized dopant atoms; this charge density has abrupt limits (step function) and the locations of these limits is easily obtained from material parameters, bias conditions using simple analysis. Whitout these considerations, obtaining quantitative solutions could only be made by numerical methods.

Historically, the depletion approximation have always been used to explain quantitatively the physical phenomena observed in p-n junctions. Nevertheless, the depletion approximation neglects several features, which are well known to occur in junction charge distributions, such as inversion layers at the less doped side, incomplete removal of free carriers on the more doped side and gradual decaying of charge density that extends away from the edge of the depletion region. These features modify not only the charge density profile, but also the eletrical field and eletrical potential profiles.

This work has as main goal to analyse the differences between the solutions obtained using numerical methods and analytical solutions provided by the depletion approximation. This is done in order to establish a limit for using this approximation with acceptable agreement.

METHODOLOGY

Looking at Figure 1, two large differences between the results obtained using depletion approximation and numerical methods can be seen.

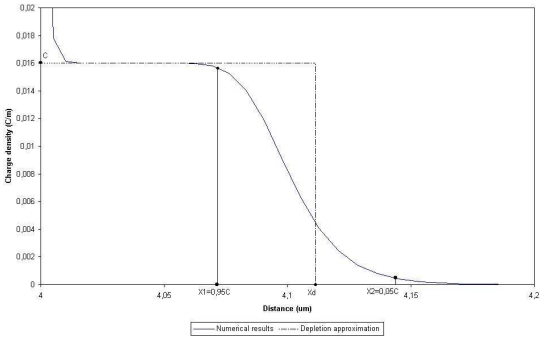

FIG.1: Charge density for n-doped (less doped) side of a assymetrically doped abrupt junction. Solid curve is the exact numerical result. Dotted curve is from depletion approximation. Marked points are parameters to determine σ.

The first of them is the gradual decaying of charge density. The second one is the high charge density present near the metalurgical junction, that leads to charge density values larger than the doping concentration, forming an inversion layer. These two phenomena, not predicted by the depletion approximation, can determine, hence, the bounds for the use of this approximation.

The method chosen for doing that consists in solving the following equation for several junctions with different values of doping and bias conditions:

$$\sigma = (x_2 - x_1)/(X_d - Xj) \qquad (1)$$

where X_1 is the point at x coordinate where the charge density presents 95% of the value determined by the depletion approximation (point C) and X_2 the point at x coordinate where the charge density presents 5% of this value. X_d is the point at x coordinate where the depletion region ends (See Figure 1). The parameter X_j represents the depth of the junction (in this case 4 μm).

To the configurations (doping and bias conditions) that present high values of σ the use of depletion approximation will not be recommendable.

A comparison between the potential calculated by the depletion approximation and the numerical methods was made by calculating the difference between them at the end of the depletion region and dividing this difference by the total potential on the junction ($[V_2 - V_1]/V_j$). Again, to the configurations that present high

values of this calculation, the use of the depletion approximation will not be recommendable.

DISCUSSION OF RESULTS

Figures 2 and 3 show the results obtained for charge density and Figures 4 and 5 for potential.

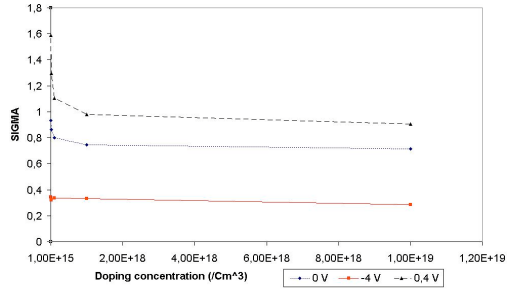

FIG.2: Values of σ for different doping concentrations an bias conditions. Symmetrical PN junction.

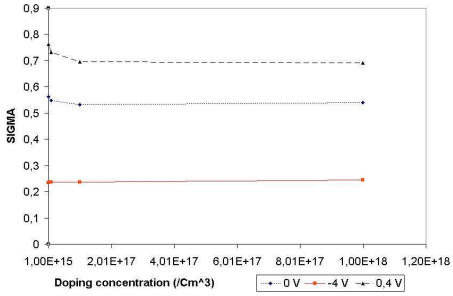

FIG.3: Values of σ for different doping concentrations an bias conditions. P^+N junction with fixed $N_A=10^{19}$ Cm^{-3}.

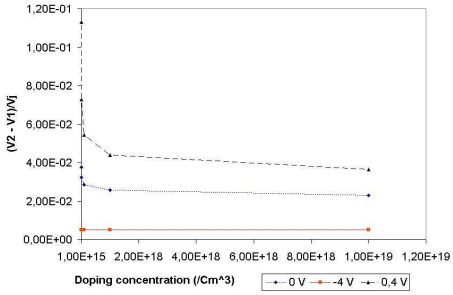

FIG.4: Relative difference (for potential) between depletion approximation and numerical solutions. Simmetrical PN junction.

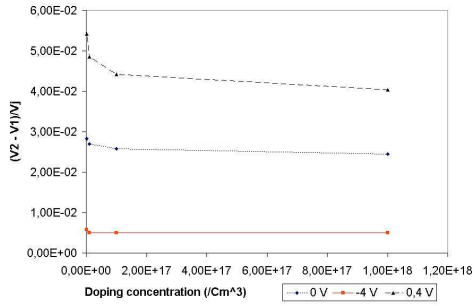

FIG.5: Relative difference (for potential) between depletion approximation and numerical solutions. P^+N junction with fixed $N_A=10^{19}$ Cm^{-3}.

By these graphs one can observe that for both, charge density and potential, higher values of doping make the depletion aproximation to be more accurate. It also occurs at reverse bias conditions. This can be explained by the fact that at reverse bias conditions, the inversion layer is reduced and charge density profile gets relatively closer to the depletion approximation due to the larger value of the depletion width. The accuracy for the potential, in this case, is around 99,5% (a very small diference), what shows that depletion approximation can be used for reverse bias condition.

For forward bias condition, the inversion layer increases (correspondig to increasing injection of minority carriers) and the charge density loses its step function characteristic and becomes relatively more gradual. The accuracy of the potential, in this case, is around 95%. This apparently small error has already great influences on the holes and electrons density (see Boltzmann's relations). Hence it is not recommendable to use the depletion approximation in this case if high accuracy is needed.

The zero bias condition presents results between forward and reverse bias. The accuracy of the potential, in this case, is around 98% which also represents great influences on the electrons and holes densities.

CONCLUSION

The depletion approximation is extremely useful in understanding the physics of junction operation and as a initial approximation for iterative numerical solutions. However, for accurate results it is only recommendable to use it when the junction is reversely biased. The incorrectness on forward and zero bias conditions may influence significatively on the electrons and holes densities. If is needed an analytical solution that does not neglect features that occur in junctions such as inversion layers and gradual decaying, one can use the results deduced by Shirts and Gordon[3]. More detailed results are shown at our site[4].

REFERENCES

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[4] www.ccs.unicamp.br/~dellalucia.