2D Numerical Modeling of Hall Effect Devices

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ABSTRACT
In this work, a proposal for a 2D numerical modeling methodology of Hall Effect semiconductor devices is presented, employing linear finite-differences. Specific optimization of the device's geometry and construction are being considered for improving sensibility in the future implementation of an Educational Integrated Circuit, using commercial CMOS fabrication processes, and the possible integration of other test structures, such as the Corbino disk.

Keywords
Numerical Simulation, Hall Effect, Finite Differences, Transport Equations.

1. INTRODUCTION
Currently, there is a technological demand for the development of integrated sensors that detect magnetic fields. The Hall Effect sensor is commonly used because it is possible to build sensors with high quality through standardized manufacturing processes. Typically, a Hall sensor may be constructed in three common types of geometries: Rectangular, Greek-cross and Diamond-shaped, as shown in Figure 1 [1]. Furthermore, these geometries are easily integrated in currently available CMOS processes.

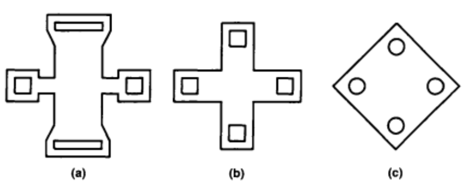

Figure 1. a)Rectangular, b)Greek-cross, c)Diamond.

Basically, a Hall Effect sensor may be represented as in Figure 2, in a 3D Cartesian frame of reference, where a semiconducting rectangular slab is exposed to an externally applied magnetic field, along the z-axis. The slab has length L and width W and is externally connected by electrodes to a voltage source and a voltage meter. Assuming that a voltage is applied to the ends of the slab along the x-dimension and a drift current is generated in this same direction, a Lorentz force, resulting from interaction of the moving charges inside the slab with the magnetic field, generates a potential difference V_H (Hall Potential) along the y-axis [2].

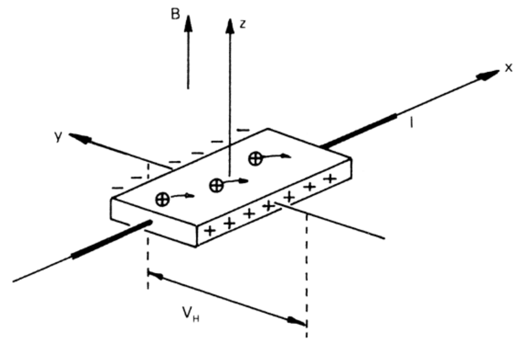

Figure 2. Hall Sensor.

This phenomenon was accidentally discovered by Edwin Hall in 1879, and was named the Hall Effect. The development of behavior models for these devices, in order to optimize its sensibility in detecting and measuring magnetic fields is an extensive area of research. The Hall sensor applications are found in many niches, from consumer devices (i.e. cell phones) to the applications in magnetic storage devices (i.e. hard drives).

2. TRANSPORT EQUATIONS
2.1 Drift Current Density
Since the Hall sensor is a semiconductor device, in the presence of external electric and magnetic fields, the Lorentz force and carrier speed are obtained from (1) and (2) respectively:

$$\vec{F} = q(\vec{E} + \vec{v}\, x\, \vec{B}) \tag{1}$$

$$\vec{v} = \mu\vec{E} + \mu^*(\vec{v}\, x\, \vec{B}) \tag{2}$$

Where $\boldsymbol{q}$ is the elementary charge, $\boldsymbol{E}$ is the electric field vector, $\boldsymbol{v}$ is drift velocity vector, and $\boldsymbol{B}$ is the magnetic flux vector, $\boldsymbol{\mu^*}$ is the Hall mobility. Solving Equation (2) for the two-dimensional case, where Hall mobility is different from the carrier mobility:

$$\begin{bmatrix} v_x & v_y & 0 \end{bmatrix} = \mu\begin{bmatrix} E_x & E_y & 0 \end{bmatrix} + \mu^*\, det \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & 0 \\ 0 & 0 & B_z \end{bmatrix} \tag{3}$$

From Equation (3) the 2D components of the drift velocity are:

$$v_x = \mu E_x + \mu^*(v_y\, B_z) \tag{4}$$

$$v_y = \mu E_y - \mu^*(v_x\, B_z) \tag{5}$$

The device is constituted essentially by n-type doped silicon. The electrons' speed vector is derived from the drift velocity equations:

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This work has been developed by the first author(s) in the scope of the undergraduate studies

$$\vec{v}_n = \mu_n \vec{E} + \mu_n^* (\vec{v}_n x\, \vec{B}) \quad (6)$$

Solving Equation (6) for the 2D case:

$$v_{nx} = \mu_n E_x + \mu_n^* (v_{ny}\, B_z) \quad (7)$$

$$v_{ny} = \mu_n E_y - \mu_n^* (v_{nx}\, B_z) \quad (8)$$

And the electric field vector is the gradient of the potential:

$$\vec{E} = -\vec{\nabla} \emptyset \quad (9)$$

Considering Equation (9), for the drift velocity, the following relations are obtained:

$$v_{nx} = -\mu_n \frac{\partial \emptyset}{\partial x} + \mu_n^* (v_{ny}\, B_z) \quad (10)$$

$$v_{ny} = -\mu_n \frac{\partial \emptyset}{\partial y} - \mu_n^* (v_{nx}\, B_z) \quad (11)$$

In which the index n indicates the n-type material. Knowing that the electrical current density is defined as the amount of current passing through a given section area, the following equations provide a simple linear relation for electron velocity in the two-dimensional case, as initially proposed:

$$\frac{\vec{J}_n}{\rho_n} = \vec{v}_n = -\mu_n \vec{\nabla} \emptyset + \mu_n^* (\vec{v}_n x\, \vec{B}) \quad (12)$$

$$\vec{J}_n = -\mu_n \rho_n \vec{\nabla} \emptyset + \mu_n^* \rho_n (\vec{v}_n x\, \vec{B}) \quad (13)$$

Solving Equation (13) for electrons:

$$J_{nx} = -\mu_n \rho_n \frac{\partial \emptyset}{\partial x} + \mu_n^* J_{ny}\, B_z \quad (14)$$

$$J_{ny} = -\mu_n \rho_n \frac{\partial \emptyset}{\partial y} - \mu_n^* J_{nx}\, B_z \quad (15)$$

Where the total electron current is given by:

$$\vec{J}_n = J_{nx}\, \hat{x} + J_{ny}\, \hat{y} \quad (16)$$

2.2 Continuity and Poisson Equations

The Poisson equation and the continuity equation are used as described below, respectively:

$$\nabla^2 \emptyset = -\rho / \varepsilon_r \varepsilon_0 \quad (17)$$

$$\nabla \vec{J} = qR = q \frac{n - n_0}{\tau_n} \quad (18)$$

Where $\emptyset$ is the electric potential, ρ is the charge density, and $\varepsilon_r \varepsilon_0$ is the permissivity of silicon. The concentration of electrons is $\boldsymbol{n}$, which depends on the equilibrium temperature [3].

The model equations [3][4] here presented may be solved using linear finite difference equations.

3. BOUNDARY CONDITIONS

To solve the proposed set of transport and field equations for specific Hall sensor geometries, it is necessary to establish the boundary conditions of the device to be modeled. In this work, two geometries are specially considered: the Greek-cross and the Corbino disk. For the Greek-cross (Figure 3), a regular Cartesian lattice is used, along the x and y directions. Potentials applied are arbitrarily attributed to frontier lattice cells, representing external sources. Small potential differences are considered to avoid nonlinear effects, such as resulting from electron velocity saturation. Low uniform magnetic fields of about 0.1 Tesla shall be considered.

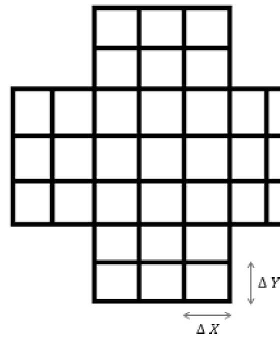

Figure 3. Greek-cross Lattice.

For the Corbino disk device geometry, the discrete lattice for solving the finite-difference equations require cylindrical coordinates. The disc is an object with radial symmetry. A potential difference is applied between the center of the disc and the outer end, as represented in Figure 4. But a numerical solution from a linear scheme can still be obtained.

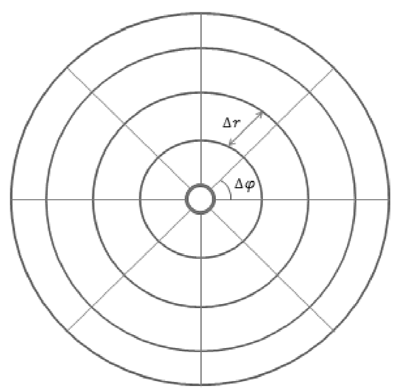

Figure 4. Corbino Disk

4. METHODOLOGY

The finite difference numerical method [5] was adopted to solve the equations of proposed Hall Effect semiconductor device. This method has become popular because it is easily applied to ordinary and partial linear differential equations. The finite difference method approximates first and second order derivatives as follows:

$$\frac{\partial f}{\partial x} = \frac{f_{(i+1,j)} - f_{(i,j)}}{\Delta x} \quad \text{1st Order Progressive Difference} \quad (19)$$

$$\frac{\partial f}{\partial x} = \frac{f_{(i,j)} - f_{(i-1,j)}}{\Delta x} \quad \text{1st Order Regressive Difference} \quad (20)$$

$$\frac{\partial f}{\partial x} = \frac{f_{(i+1,j)} - f_{(i-1,j)}}{2\Delta x} \quad \text{2nd Order Central Difference} \quad (21)$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{f_{(i+1,j)} - 2f_{(i,j)} + f_{(i-1,j)}}{\Delta x^2} \quad \text{2}^{\text{nd}} \text{ Order Central Difference} \qquad (22)$$

From the Poisson's equation for the electric potential and taking into account the previously presented transport and field equations that describe the behavior of a Hall Effect semiconductor device, and the continuity equation in steady state form, the following discrete finite-difference linear equations are proposed [5].

$$v_{nx(i,j)} = -\mu_n \frac{\emptyset_{(i+1,j)} - \emptyset_{(i-1,j)}}{2\Delta s} + \mu_n^*(v_{ny(i,j)}\, B_z) \qquad (23)$$

$$v_{ny(i,j)} = -\mu_n \frac{\emptyset_{(i,j+1)} - \emptyset_{(i,j-1)}}{2\Delta s} - \mu_n^*(v_{nx(i,j)}\, B_z) \qquad (24)$$

$$\emptyset_{(i,j-1)} + \emptyset_{(i,j+1)} - 4\emptyset_{(i,j)} + \emptyset_{(i+1,j)} + \emptyset_{(i-1,j)}$$

$$+ \frac{\rho_{(i,j)}}{\varepsilon_r \varepsilon_0} = 0 \qquad (25)$$

$$\rho = -nq \qquad (26)$$

$$J_{nx(i+1,j)} - J_{nx(i-1,j)} + J_{ny(i,j+1)} - J_{ny(i,j-1)}$$

$$-2\Delta s \rho_{(i,j)} + \frac{2\Delta s q n_{(i,j)}}{\tau_n} = 0 \qquad (27)$$

The current density may be obtained from:

$$J_{nx(i,j)} = \rho_{(i,j)}\, v_{nx(i,j)} \qquad (28)$$

$$J_{ny(i,j)} = \rho_{(i,j)} v_{ny(i,j)} \qquad (29)$$

5. CONCLUSION

A linear 2D model for solving transport and field equations in Hall Effect devices has been presented. A complete set of linear finite-difference equations has been proposed for obtaining charge and current densities, as well as electric potentials in such a device, under a uniform perpendicular magnetic field, and with arbitrary boundary conditions. A numerical solver is being developed and implemented using the free software SCILAB (www.scilab.org), which has similar functions and resources as found in MATLAB (www.mathworks.com).

6. REFERENCES

[1] Ramsden, E.,2006. *Hall Efect Sensors Theory and Practice.* Newnes, Ed. Elsevier, 26.

[2] Popovic, R. S. et al, *Integrated Hall-effect magnetic Sensors*, Sensors and Actuators A 91 (2001).

[3] Sze, S.M. and Kwok, K., 2006, *Physics of Semiconductor Device. Wiley-Interscience, Third edition.*

[4] Joerg, W. B.and Nathan, A., 1992, *Albertina: A Program Package to Numerically Model Carrier Transport in Magnetic Field Sensors.* University of Alberta, Edmonton, Alberta, Canada.

[5] Chapra, S. C. and Canale, R. P., *Métodos Numéricos para Engenharia*, Ed. McGraw Hill.