Proposal of a Single-Phase Wattmeter Based on Analog Signal Multiplication Employing a Switched Capacitor Modulator

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***Abstract*—This work discusses an approach for analog signal multiplication that uses a switched capacitor circuit, associated to a pulse width modulation strategy. The topology is described, followed by a mathematical analysis. This demonstrates that it is possible, after a proper filtering, to produce a signal proportional to the product of the two input signals, as well as the average value of this product. If one considers the input signals as a voltage and a current into a load, then the average value of the multiplier output signal is proportional to the consumed power, what justifies the use of the considered structure for implementing a single-phase wattmeter. Experimental results are shown, proving the validity of the approach for different input signals.**

Keywords—Analog multiplication; switched capacitor; pulse width modulation; Programmable System on Chip; wattmeter

I. INTRODUCTION

Analog multipliers have a large number of applications in areas such as telecommunications, control, instrumentation, measurement and signal processing [1].

The most encountered signal multiplication structure is the Gilbert cell, which can be built with few transistors. The basic concept of analog multiplication employing Gilbert cells is based on the exponential characteristic of the bipolar junction transistors, but it is also feasible in the CMOS technology [2]. The drawback of this approach is the need for pre-distortion circuits, in order to multiply signals whose order of magnitude is higher than the transistors thermal voltage, V_T (typically 26 mV for a room temperature of 25°C).

Another method consists on employing logarithmic and exponential amplifiers, which also use the exponential feature of bipolar transistors for making the multiplication operation. The major drawback in this technique is that the formed circuit is capable of only the first quadrant multiplication [3].

The objective of this work is to propose a single-phase wattmeter, which uses a switched capacitor circuit to do an analog signal multiplication, where the two input signals are related to the voltage and current into a load.

The topologies based on switched capacitors are used to perform several analog functions. Moreover, the structures based on switched capacitors are proper for VLSI circuits integration. The main reasons for it are: (i) compatibility with most of the CMOS fabrication processes; (ii) accuracy based on the capacitances rates, instead of the absolute values of the capacitors (which can be impaired due to fabrication process variations); and easy migration to recent CMOS Technologies [4].

The paper is organized as follows: Section 2 describes the topology, also including a brief mathematical analysis. In the Section 3, the system implementation is described, and experimental results are shown. The main conclusions and future works are presented in the Section 4.

II. THE PROPOSED TOPOLOGY

The proposed analog signal multiplication topology uses switched capacitor (SC) amplifiers in order to make an analog modulation.

The non-inverting SC amplifier is presented in Fig. 1. The switches are driven by logic signals with complementary, non-overlapped phases, $\phi 1$ and $\phi 2$. The gain for the non-inverting topology is C_A/C_F [5].

The inverting configuration is implemented simply by swapping the phases that drive the switches inside the dotted line. In this case, the gain is $-C_A/C_F$. Then, its magnitude is the same as the one found for the non-inverting configuration.

If a given circuit has the capability to select the SC amplifier configuration, it is possible to apply a modulation strategy on a voltage input signal. In this work, this topology, shown in Fig. 2, will be named *SC modulator*. In this circuit, the logic signal *Sign* swaps the position of the switches between terminals 1-2/3-4 (*Sign* = 1) and 1-4/3-2 (*Sign* = 0). If we can assume $C_A = C_F$, for simplicity, than the circuit gain will alternate between +1 and -1, being determined by *Sign*.

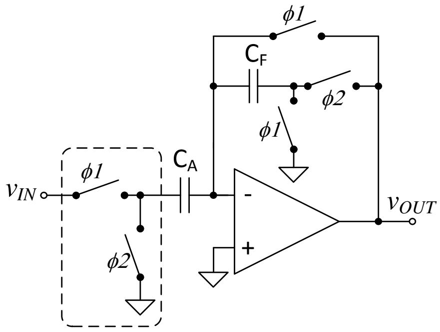

Fig. 1. Non-inverting SC amplifier.

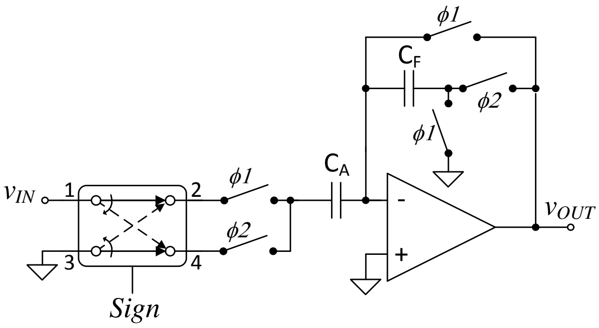

Fig. 2. SC modulator.

A. System description

The system that uses the SC modulator for implementing the signal multiplication is illustrated by the block diagram shown in Fig. 3. Signal v_{IN1} is compared to a triangular carrier signal v_c, resulting in a pulse width modulated (PWM) signal, employed to select the amplifier configuration in the SC modulator. This, in turn, receives at its input the signal v_{IN2}. The resulting signal v_{MOD} will contain a component that is proportional to the product of v_{IN1} and v_{IN2}, and high frequency components that are inherent to the PWM signal.

The low-pass filter LPF1 is used to give the product of the signals, while filter LPF2 is used to extract the average value of the product. This, in turn, will be digitized by an analog-to-digital (A/D) converter, ADC, and exhibited on a liquid crystal display (LCD).

A circuit presented in [6] performs the multiplication of two signals by a similar principle, but employs two different continuous-time amplifiers, whose outputs are swapped by analog switches, driven by a PWM signal. The strategy presented in [7] makes use of the topology of Fig. 2 in order to make the analog multiplication.

If v_{IN1} and v_{IN2} are interpreted, respectively, as the voltage v_L and the current i_L flowing into a load, then the output of LPF1 will yield a signal proportional to the instantaneous power *p(t)* delivered to the load, while the output of LPF2, digitized by the ADC, will give an indication proportional to the average power consumed by the load, *P*, which is expressed by the definition,

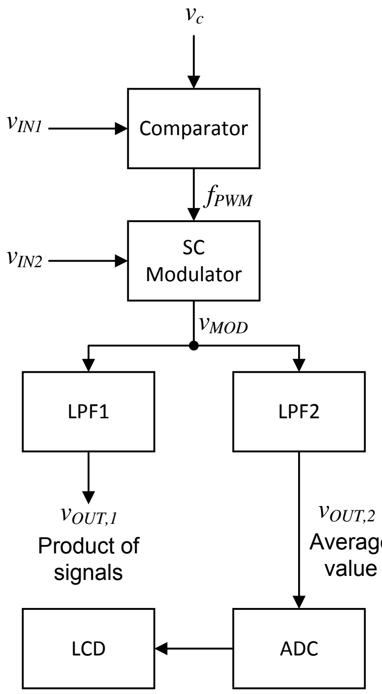

Fig. 3. Block diagram that illustrates the signal multiplication using the SC modulator.

$$P = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T v_L(t)\cdot i_L(t)dt\,. \qquad (1)$$

B. Mathematical analysis

An analytical solution that lets to identify the harmonic components of a PWM signal can give a mathematical basis for understanding the filtering process that produces the desired signals.

This signal can be modeled using the double Fourier series [8]. For a PWM signal f_{PWM} that varies between -1 and +1, originated from the comparison between a triangular waveform and a sinusoidal waveform with angular frequencies of ω_c and ω_0, respectively, the resulting expression is

$$\begin{aligned} f_{PWM}(t) &= M\cos(\omega_0 t) + \\ &\sum_{m=1}^{\infty}\left[\frac{4}{m\pi}J_0\left(m\frac{\pi}{2}M\right)sen\left(\frac{m\pi}{2}\right)\cos(m\omega_c t)\right] + \\ &\sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}\left[\frac{4}{m\pi}J_n\left(\frac{m\pi M}{2}\right)sen\left(\frac{\pi}{2}(m+n)\right)\right. \\ &\left.\times\cos(m\omega_c t + n\omega_0 t)\right]. \end{aligned} \qquad (2)$$

The parameter *M* is the modulation index, given by

$$M = \frac{A_{0(1)}}{A_c}, \qquad (3)$$

Where $A_{0(1)}$ e A_c are the magnitudes of the reference (v_{IN1}) and carrier (v_c), respectively. $J_n(\theta)$ is the *n*-th order Bessel function [9].

It can be noted that the first term in (2) has the same frequency as the reference signal (v_{IN1}), whose amplitude is implicitly represented by the modulation index M. The following summations are related to the high frequency components.

The output of the SC modulator is given by the multiplication of the PWM signal and another signal, identified as v_{IN2}.

$$v_{MOD}(t) = f_{PWM}(t) \times v_{IN2}(t) \tag{4}$$

Applying the modulated signal v_{MOD} into the LPF1 filter, the resulting signal will be

$$v_{OUT,1}(t) = M\cos(\omega_0 t) \times v_{IN2}(t)\,. \tag{5}$$

If v_{IN2} is also sinusoidal and with frequency ω_0, amplitude $A_{0(2)}$ and phase $-\theta$ related to v_{IN1}, then

$$\begin{aligned} v_{OUT,1}(t) &= M\cos(\omega_0 t) \times A_{0(2)}\cos(\omega_0 t - \theta) = \\ &= \frac{MA_{0(2)}}{2}\left(\cos(\theta) + \cos(2\omega_0 t - \theta)\right)\,. \end{aligned} \tag{6}$$

At the output of LPF2, we can obtain the average value of the modulated signal. For v_{IN1} and v_{IN2} considered above, it yields

$$v_{OUT,2}(t) = \frac{MA_{0(2)}}{2}\cos(\theta)\,. \tag{7}$$

C. *Filter design*

The two low-pass filters were designed with aid of Texas Instruments *FilterPro* software. The parameters are shown in Table I. It can be noted that the cutoff frequency of LPF2 is quite lower than the fundamental frequency of the multiplied signals, due to the need of extracting the DC component of the modulated signal.

TABLE I. PARAMETERS OF LOW-PASS FILTERS LPF1 AND LPF2.

Parameter	**Filter**	
	LPF1	***LPF2***
Type	Butterworth	Bessel
Order	4	5
Gain	0 dB	0 dB
Passband ripple	1 dB	1 dB
Passband frerquency	1 kHz	6 Hz
Atenuation	-3 dB	-3 dB
Reject band atenuation	-45 dB	-45 dB
Reject band	5 kHz	30 Hz

III. PROTOTYPE IMPLEMENTATION AND EXPERIMENTAL RESULTS

For implementation of the prototype, the development kit *CY8CKIT-001*, from Cypress, was used. It has a PSoC (Programmable System on Chip) CY8C28 Family Processor, based on the Cypress PSoC1 microcontrollers. This technology is based on an 8-bit microprocessor unit, and contains programmable digital and analog blocks, which can be configured by means of its IDE (Integrated Development Environment), the *PSoC Designer.*

The employed chip has six analog, continuous-time blocks, suitable for implementing different types of amplifiers and comparators. Besides the continuous-time blocks, it has ten SC blocks [10], and one of them was used to build the SC modulator. The SC blocks are fully configurable by software, and can build several analog processing structures.

The LPF1 filter could be embedded in the PSoC, using SC blocks. However, the implementation of LPF2 filter using SC blocks would not be satisfactory, due to its low cutoff frequency. The option was to implement both filters externally to the PSoC, using operational amplifiers suited for 5-V single supply operation. The common-mode voltage (the circuit analog ground) was 2.5 V, obtained from internal reference of the PSoC chip and distributed along all the circuit.

The experimental results were obtained using function generators as stimuli sources, and are presented as follows for two cases. In both of them, the carrier was a triangular waveform, with frequency and amplitude of 10 kHz and 2 V, respectively.

A. *In-phase sinusoidal waveforms*

In this first case, the two multiplied signals are in phase, with amplitude of 1 V and frequency of 60 Hz. So, the modulation index is $M = 0.5$.

The modulator output is shown in Fig. 4, while the outputs of the filters are exhibited in Fig. 5. From the measurements taken with a digital oscilloscope, the output of LPF2 yielded a voltage close to 2.75 V, corresponding to an effective average voltage of 250 mV. This result agrees with (7).

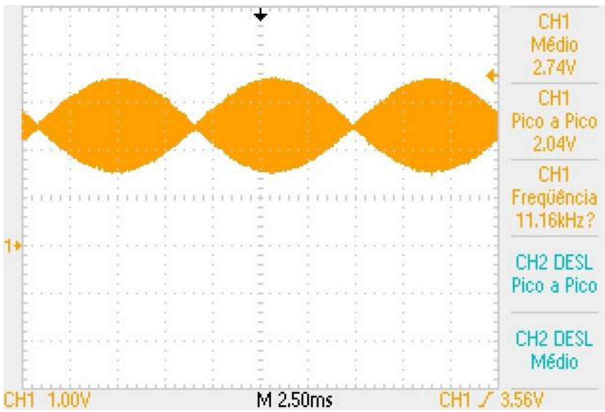

Fig. 4. Modulator output for two in-phase sinusoidal signals. Vertical: 1 V/div; Horizontal: 2.5 ms/div.

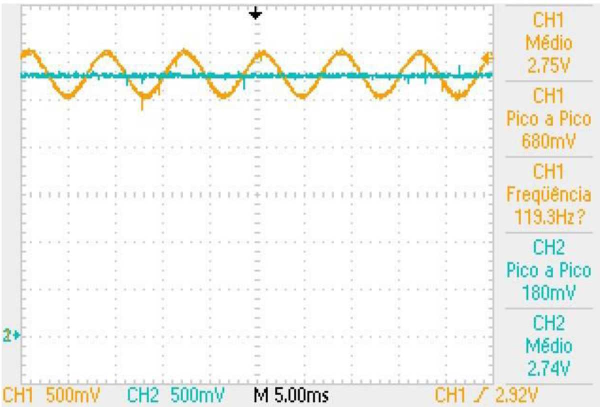

Fig. 5. Output of the filters (in-phase signals). Channel 1: output of the LPF1; Channel 2: output of the LPF2 (average value). Vertical: 500 mV/div; horizontal: 5 ms/div.

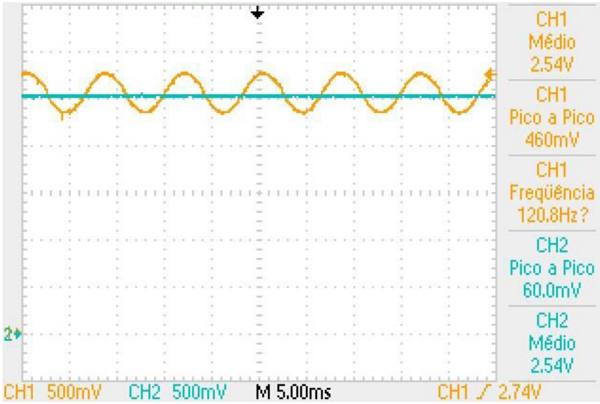

Fig. 6. Output of the filters (out of phase signals). Channel 1: output of the LPF1; Channel 2: output of the LPF2 (average value). Vertical: 500 mV/div; horizontal: 5 ms/div.

B. *Out of phase sinusoidal waveforms*

For two sinusoidal signals with amplitude 1 V, frequency 60 Hz and phase displacement close to 80^o, the output of the filters are shown in Fig 6. The average voltage value was close to 2.54 V, corresponding to an effective average of approximately 40 mV. This was an expected result, since these signals represent the voltage and current into a load with a low power factor.

IV. Conclusions

This work described a topology capable of making the four-quadrant, analog multiplication of two signals by means of a switched capacitor block, which was denominated SC modulator. If the modulator output is properly filtered, it is possible to obtain the waveform of the product of the signals, as well as its average value.

The system can be used for the development of an experimental single-phase wattmeter. The only step that implies some digital processing is the A/D conversion of the average value filter, which is a DC value, thus relaxing the A/D converter sampling rate requirements.

Therefore, the system has a low computational cost, whose experimental results demonstrate the validity and feasibility of the proposal. The adopted platform for implementing the modulator is based on the PSoC1 microcontrollers, which have several digital and analog programmable modules, among which are the switched capacitor blocks. They can be fully configured by software.

The A/D conversion stage also uses analog and digital blocks of the PSoC, and is cited as a future work. The result will be shown in a LCD included in the development kit. Moreover, the input interfaces for isolating/scaling the line voltage levels and converting current signals into appropriate voltage signals are needed in order to build a practical wattmeter based on this approach.

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