Modified Limiter Applied to Crest Factor Reduction Based on Non-uniformly Sampled Look-up Tables

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***Abstract*— To improve the wireless transmitters, a crest factor reduction (CFR) technique known as limiting and filtering technique can be employed to reduce the peak-to-average power ratio (PAPR) of the complex envelope signal. From the literature, a significant PAPR reduction can be achieved by applying a nonlinear constrained optimization tool in a CFR technique, using finite impulse response (FIR) filter and mainly infinite impulse response (IIR) filter, with a hard-clipping limiter, where the IIR filter instability issues have been handled. Also, a fixed-point arithmetic description and a hardware realization were done for this CFR technique, using uniformly sampled linearly interpolated look-up tables (LUT). This work proposes a different limiter for the CFR technique to obtain a greater PAPR reduction and a non-uniformly sampled linearly interpolated LUT implementation, where these LUTs will replace the square root and reciprocal functions. From the case study simulations, which used a WCDMA envelope complex test signal, a larger PAPR reduction of 0.8 dB was achieved with the proposed limiter and non-uniformly sampled LUTs compared to the hard-clipping limiter and uniformly sampled LUTs, which both using 2 LUT addressable bits for square root and reciprocal functions.**

Keywords*— *Hard-clipping limiter, PAPR reduction, nonlinear optimization, linear interpolation, look-up tables.

I. Introduction

The power consumption is one of the greatest questions in wireless system projects. There is an aim to achieve high efficiency in these systems, especially in who drains the most in radio-frequency (RF) circuits, the power amplifier (PA) [1]. Due to its non-linear characteristics, the PA introduces some distortions on the signal. Nevertheless, for the RF circuits, the standard obligates them to maintain certain linearity levels, avoiding that these distortions do interfere significantly in near channel signal transmission. Therefore, in the PA project, there is a compromise between efficiency and linearity [2]. Although there are standards for the linearity, the distortion tolerable margin can be exploited willing to develop the transmitter efficiency though the peak-to-average power ratio (PAPR) reduction [3]. The crest factor reduction (CFR) can be employed to reduce the PAPR value by clipping the signal peaks and increasing the average output power [4].

In [5], a non-linear constrained optimization of CFR technique was presented, using a finite impulse response (FIR) filter and a hard-clipping limiter. In [6], this optimization is extended by replacing the FIR filter with an infinite impulse response (IIR) filter and placing a new constraint to avoid the instability possibly conditioned by the IIR poles. In [7], the fixed-point arithmetic description and FPGA implementation of CFR technique were presented, for both digital filters, using linearly interpolated look-up tables (LUT) obtained from an uniform sampling of the original function. This work aims to propose a new model for the CFR technique limiter and a CFR realization employing LUTs originated from non-uniform function sampling.

The section II presents the CFR technique and the limiters, the section III presents the digital linear filters, the section IV shows the CFR nonlinear optimization, the section V the uniformly and non-uniformly sampled LUTs, the section VI the MATLAB results and the section VII the conclusions.

II. Limiting and Filtering PAPR Reduction Technique

Among the CFR techniques, who aim to reduce the complex signal PAPR value, a popular one is the limiting and filtering technique. It can be represented by a two-cascade block set: a limiter and a digital linear filter [6-7].

The limiter is the first CFR block. Its goal is to clip the complex values whose amplitudes surpass a pre-established value. In this work, it is studied a hard-clipping limiter and a proposed limiter. The Fig. 1 presents output amplitudes in relation to input amplitudes for the two limiters, where $u[n]$ and $s[n]$ are the limiter input and output signals, respectively. The Figs. 2 and 3 show the block diagrams of the hard-clipping and proposed limiters, respectively, where a_1 and a_2 are line angular coefficients, b_2 is line linear coefficient, M is the signal amplitude and $\exp\{j\theta\}$ is the input signal phase. Note that, from the Figs. 2 and 3, there are no changes in the signal phase because these limiters only modify the signal amplitude. Also note that the hard-clipping limiter only has one coefficient, the clipping factor L, and only modifies the amplitude signal in the saturated region. Otherwise, the limiter from the Fig. 3 presents a gain in the linear region and has three coefficients: the clipping factor L and the coordinates, Tx and Ty, of the transition point between the first two lines.

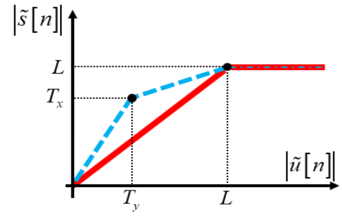

Fig. 1. Limiter output amplitudes in relation to input amplitudes.

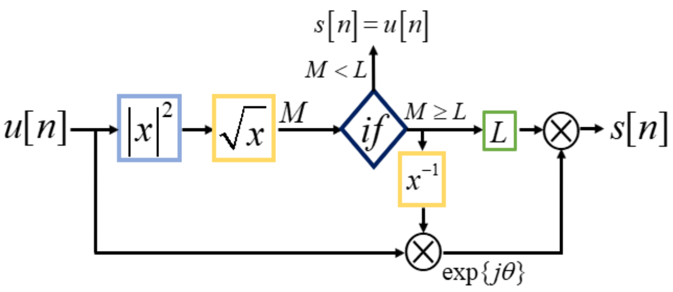

Fig. 2. Hard-clipping limiter block diagram.

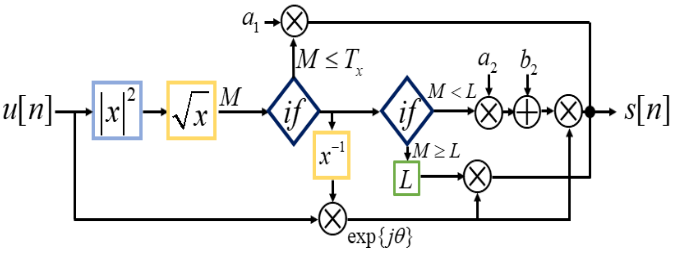

Fig. 3. Proposed limiter block diagram.

Therefore, the proposed limiter seeks to clip the signal peaks and simultaneously to increase the average output power, so that it could reduce more the PAPR compared to the hard-clipping limiter. However, these limiters generate distortion levels inside and outside the main channel. It is worth to mention that the hard-clipping limiter can be seen as the particular case of the proposed limiter, when the transition point coordinates, *Tx* and *Ty*, present the same values.

III. Linear Digital Filters

The second CFR block is the linear digital filter. Its objective is to reduce partially the amount of distortions generated by the CFR first block [6-7]. The consequence of the filter operation is that the signal peaks clipped by the limiter are partially restored, hence increasing the PAPR value. So, there is a negotiation between the PAPR reduction and the distortions levels in the channel.

In here, two digital filter classes are studied: finite impulse response (FIR) and infinite impulse response (IIR).

A. FIR Filter

The FIR filter is a digital filter class where the output signal, in a determined time instant, is represented by a combination formed by the input signal, both in present time and past times, and by filter coefficient set [8]. The FIR filter mathematical modeling is given by:

$$y[n]=\sum_{m=0}^{M} g_m s[n-m], \tag{1}$$

where $s[n]$ and $y[n]$ are input and output samples, respectively, g_m the FIR filter coefficients and M the amount of past samples. A FIR important propriety is the immunity to instability due the feedback absence.

B. IIR Filter

The IIR filter is a digital filter class more general than the previous one. For a certain instant, its output signal is formed by a combination of the input samples, in present and past times, like the FIR filter, of the output samples in past times and of a filter coefficient set [8]. Unlike the FIR filter, the IIR filter is vulnerable to instability condition due the negative feedback propriety, where this condition is achieved when the IIR filter possesses at least one pole whose amplitude is not lower than one. From the constant coefficient linear difference equation, the IIR filter mathematical representation is showed as follows [9]:

$$y[n]=\sum_{m=0}^{M} c_m s[n-m]-\sum_{k=0}^{N} d_k s[n-k], \tag{2}$$

where c_m and d_k are the coefficients associated to the input and output samples, respectively, M and N the amount of past samples related to the input and output samples, respectively.

IV. CFR Parameter Identification

In this work, based on a nonlinear constrained optimization, it is selected the CFR coefficients, like the clipping factor L from the Fig. 2, the transition point coordinates *Tx* and *Ty* from the Fig. 3, either the FIR coefficients g_m from (1) and the IIR coefficients c_m and d_k from (2). This nonlinear optimization aims at reducing the complex signal PAPR value and simultaneously not surpassing the maximum tolerable distortion levels, which are quantified by the following metrics: error vector magnitude (EVM) and adjacent channel power ratio (ACPR). EVM is a metric which works with time-domain samples and measures the distortions placed inside and outside the main channel. EVM is represented as:

$$EVM=\frac{\sqrt{\sum_{n=1}^{N_T}\left|y[n]-u[n]\right|^2}}{\sqrt{\sum_{n=1}^{N_T}\left|u[n]\right|^2}}\cdot 100\%, \tag{3}$$

where $u[n]$ and $y[n]$ are the output and input CFR samples, respectively, and N_T the number of available samples. In another hand, the ACPR works with frequency-domain signals and quantifies the amount of distortion in the adjacent channels. ACPR is given by:

$$ACPR=10\log_{10}\left[\frac{\int_{adj}\left|Y(f)\right|^2 df}{\int_{main}\left|Y(f)\right|^2 df}\right], \tag{4}$$

where *Y*(*f*) indicates the CFR output signal, described in the frequency-domain, and the indexes *adj* and *main* indicate the main and adjacent channels, respectively.

In addition, for the proposed limiter, a new constraint is set to impose that the intermediary line must have a non-negative linear coefficient value (b_2), ensuring that there is not any attenuation in the non-clipped signal. For the IIR filter case, to escape from the instability condition, a new constraint is added to guarantee that all IIR filter poles do not present amplitudes equal or greater than one. The mathematical modeling for the constrained optimization algorithm is:

$$\min_{x} PAPR(x) \text{subject to} \begin{cases} EVM(x) \le MAX_{EVM} \\ ACPR(x) \le MAX_{ACPR} \\ b_2(x) \ge 0 \\ \max\left\{\left|Vector_{poles}(x)\right|\right\} < 1 \end{cases}, \quad (5)$$

where MAX_{EVM} and MAX_{ACPR} are the maximum tolerable values for the EVM and ACPR metrics, respectively, $Vector_{poles}$ a vector containing all the IIR filter pole values, and x the optimization CFR coefficient vector, where these coefficients are the clipping factor L, the transition point coordinates Tx and Ty when used the proposed limiter, either the FIR or IIR filter coefficients. Since both the objective and constraint functions are non-linearly dependable of the optimization variables, a non-linear tool is required for this constrained optimization. In non-linear algorithms, the optimization variable set may be trapped into local minima, where this depends of the initial value set that must be entered by the algorithm user.

Due to instability condition and a higher number of coefficients for the IIR filter case, the expectation is that the optimizer performance will be more sensitive to IIR initial guesses [6-7].

V. Linearly Interpolated Look-up Tables

Targeting a CFR fixed-point arithmetic description, in the limiter the square root and reciprocal functions are replaced by linearly interpolated look-up tables (LUTs), who are capable to reproduce mathematical complex functions. In this work, the contents recorded in the LUTs are the angular (α) and linear (β) coefficients [10]. The LUT output is given by:

$$L_{out} = \alpha \cdot L_{in} + \beta, \quad (6)$$

where Lin is the LUT input and the coefficient values are:

$$\alpha = \frac{y_2 - y_1}{x_2 - x_1}, \quad (7)$$

$$\beta = y_1 - x_1 \cdot \frac{y_2 - y_1}{x_2 - x_1}, \quad (8)$$

where x_1 and x_2 are original input function points, and y_1 and y_2 original output points. Here, two methods for linear interpolation are studied: one with the uniformly sampled of the original input function and another where the original input function sampling is performed in a non-uniform way.

VI. Matlab Simulation Results

In this section, the limiters presented in the Figs. 2 and 3, and both FIR and IIR filters in (1) and (2), are applied to the PAPR reduction of a test signal. This test signal is a 2,048 time-domain sequence of a WCDMA complex envelope, with a sampling frequency of 61.44 MHz, a bandwidth of 3.84 MHz and whose PAPR value is 9.7 dB.

The constrained nonlinear optimization is done in MATLAB by applying an interior point algorithm with a double precision floating-point arithmetic [11]. In relation to the nonlinear optimization initial guesses, for the limiters, the clipping factor L was chosen from a closed interval between 0.3 and 0.7, the transition point x-coordinate from the closed interval between 0.1 and 0.5, the transition point y-coordinate from the closed interval between 0.2 and 0.6. In doing the initial guesses for the proposed limiter coefficients, it is important that x-coordinate value be lower than the clipping factor in order to help in the constrained optimization convergence. About the linear digital filter initial guesses, for the FIR filter, its initial coefficients were selected randomically from an open interval within 0 to 1. Meanwhile, for the IIR filter, its initial coefficients were copied from a Butterworth low-pass approximation, which is given by [12]:

$$\left|H(\omega)\right| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}}, \quad (9)$$

where n is the filter approximation order and ω_0 the cutoff frequency. Here, n was set to 5 and ω_0 chosen between 12.5 Mrad/s and 14.1 Mrad/s.

As done in [5-7], the 3GPP standard was adopted. The EVM and ACPR maximum acceptable values are 17.5% and -45 dB, respectively. For ACPR calculus, the adjacent channel has a bandwidth of 3.84 MHz whose center is 5 MHz from the main channel center.

The Table I presents the PAPR reduction provided by the optimized CFR realization based on the original square root and reciprocal functions, with either hard-clipping or proposed limiter, and either FIR or IIR filters. From the Table I, it can be observed that for both FIR and IIR filters cases, the proposed limiter achieved a greater PAPR reduction which is attributed to the output average power rise.

TABLE I. OPTIMIZED CFR SIMULATION RESULTS

CFR Limiter	CFR Filter	PAPR reduction
Hard-Clipping	FIR	2.2 dB
	IIR	3.6 dB
Proposed	FIR	2.5 dB
	IIR	3.8 dB

The Fig. 4 presents output amplitudes in relation to input amplitudes for optimized hard-clipping and proposed limiters, using both FIR and IIR filters, respectively. From the Fig. 4, it can be noticed that the clipping factor for the CFR with IIR filter shows a lower value than for the FIR filter. Hence, the CFR realization with IIR filter clipped more the signal peaks. For both FIR and IIR filter cases, the proposed limiter showed a gain in the linear region and a clipping factor greater than the hard-clipping factor. As observed in the Table I and in Fig. 4, the CFR with proposed limiter achieved a bigger PAPR reduction, although it clipped less the signal peaks due the average output increase attributed to the linear region gain of the proposed limiter.

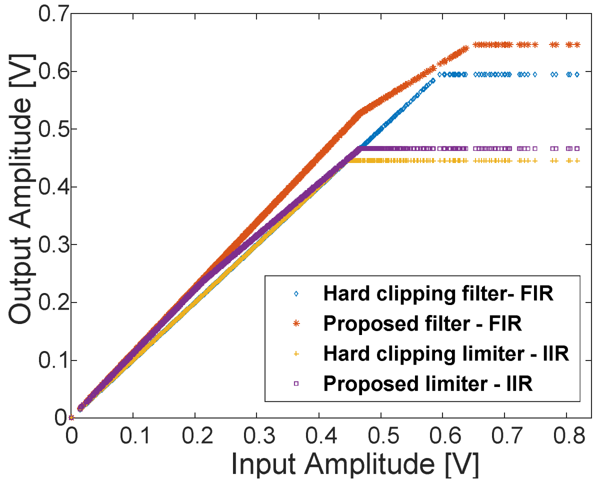

Fig. 4. Limiter amplitude transfer characteristics.

The Table II indicates the PAPR reduction, in relation to the number of LUT addressable bits, with the hard-clipping limiter and uniformly sampled linearly interpolated LUTs and the Table III shows the same, but with the proposed limiter and non-uniformly sampled linearly interpolated LUTs. These LUTs replaced the square root and reciprocal functions. From the Tables II and III, it can be noticed that, for the same number of LUT addressable bits, the CFR with proposed limiter and non-uniformly sampled LUTs presented a greater PAPR reduction than the CFR with hard-clipping limiter and uniformly sampled LUTs. In addition, for the same PAPR reduction, the CFR realization from Table III required less LUT addressable bits, for both FIR and IIR filters.

TABLE II. RESULTS FOR CFR WITH HARD-CLIPPING LIMITER AND UNIFORMLY SAMPLED LUTS

CFR Filter	Number of LUT addressable bits		PAPR reduction
	Square root function	Reciprocal function	
FIR	2	2	1.5 dB
	5	5	2.2 dB
IIR	3	2	2.7 dB
	4	6	3.6 dB

TABLE III. RESULTS FOR CFR WITH PROPOSED LIMITER AND NON-UNIFORMLY SAMPLED LUTS

CFR Filter	Number of LUT addressable bits		PAPR reduction
	Square root function	Reciprocal function	
FIR	2	2	2.2 dB
	4	3	2.5 dB
IIR	3	2	3.5 dB
	3	3	3.7 dB

VII. Conclusions

In this work, a CFR realization with a proposed limiter and non-uniformly sampled linearly interpolated LUTs was presented. Both FIR and IIR filters were applied to the limiting and filtering technique. The CFR nonlinear constrained optimization, with either hard-clipping or proposed limiter, was done. From the simulations applying a WCDMA complex envelope test signal, the optimized CFR with the proposed limiter showed a greater PAPR reduction than the CFR with hard-clipping filter. When replacing the square root and reciprocal functions by uniformly and non-uniformly sampled interpolated LUTs with the same number of LUT addressable bits, a lower PAPR value at CFR output was achieved with the non-uniformly sampled method. Therefore, a greater PAPR reduction can be achieved by combining the proposed limiter and the non-uniformly sampled linearly interpolated LUTs.

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