Two-tone Analysis Methods Applied to Simulation of Electronic Circuits

Caio G. Natalino, Dhessica M. S. de Moura and Eduardo G. Lima
Group of Integrated Circuits and Systems (GICS) – Electrical Engineering Department
Federal University of Paraná, Curitiba, Brazil

Abstract — **This article discusses some of the main analysis methods of electronic circuits stimulated to two signals with different frequencies, detailing their main characteristics and operation mode. Four methods of analysis are studied and their results are compared in order to prove their functioning and to validate them for two-tone circuits. Two power amplifier (PA) topologies are used as test circuits and harmonic orders and equal intermodulation are adopted in the simulations. The results of the simulations in Matlab show that the artificial frequency mapping has greater efficiency in two-tone circuits analysis in comparison to the studied methods, since it covers cases of small and large signals and demands less processing with less complexity.**

Keywords — Two-tone Circuits, Large Signals Analysis, Transient Simulation, Harmonic Balance, Artificial Frequency Mapping, Periodic AC Analysis.

I. INTRODUCTION

Before manufacturing, electronic circuits must be tested in a scenario as close as possible to the intended application, using circuit models and dedicated software [1]. It is from this moment that the designer will analyze the expected operation of the project, properly dimension the components and collect important results before the physical construction of the product or to make changes in the circuit. The non-linearity of the circuits is an important characteristic to be considered in these analyzes because it interferes directly in their operation. In small-signal power amplifiers (PAs), for example, non-linearity is responsible for the degradation of system performance and should be minimized, but in other circuits, as frequency multipliers, non-linearities are used in their circuit elements [2].

Currently, there are several softwares in the market that allow the simulation of electrical circuits depending on their application. They simulate a circuit from the construction of a system of differential equations that represent it and execute the solution of this system with time discretization methods [3]. It is observed that, depending on the topology of the circuit and its application condition, some methods become more efficient while others become unaccurate, in some cases not converging in their iterations [4].

This work, therefore, contributes to the study and application of two-tone circuit analysis methods for a PA test with the topologies envelope tracking (ET) and envelope elimination and restoration (EER) [5], proving the operation of the chosen simulations, from the tests of the mathematical methods, in small and large signal applications. Section II presents the PA used and its architecture, Section III addresses the small signal analysis, Section IV discusses the large signals analysis methods, while Section V reports the simulation results and Section VI presents the final conclusions.

II. POWER AMPLIFIER

The discontinuous behavior of the power amplifiers by alternating between different gain modes causes distinct nonlinearities to be observed in these circuits, the nonlinearities related to the power gain compression mechanisms that occur in the RF domain and those associated with the discontinuities that occur in the domain of the baseband [1]. One of the envelope-derived amplitude modulation techniques similar to the supply voltage of a conventional linear RF amplifier is the Envelope Tracking, which the control voltage does not replicate the envelope signal requiring high accuracy [5]. The possibility of not so precise values for the circuit supply voltage helps in the dynamic range limitations observed in other configurations.

Another PA topology is the Envelope Elimination and Restoration (EER), which was the standard method used in the modulation of AM transmitters. It is a logical alternative for high-level amplitude modulation [5].

To investigate the two-tone analysis methods, this work uses the circuit schematic of Fig. 1 [1] since it was possible to apply the two desired PA topologies for the tests and simulations, addressing small and large signals cases. For this circuit, the voltage-controlled current source described by (1) brings non-linearity to the system, where Isat and Vsat indicate the current and voltage saturations, respectively, sign( ) is the function that returns the voltage signal supplied by the independent voltage source Vs and s is the damping factor of the curve, with lower values of s indicating greater smoothness of the curve.

$$f_{NL}\left[V_A\right] = \frac{I_{sat}\,sign\left(V_A\right)}{\left(1+\left(\frac{V_{sat}}{\left|V_A\right|}\right)^{s}\right)^{\frac{1}{s}}} \tag{1}$$

The parameters of this nonlinear function are set to Isat = 0.1 A, Vsat = 1.8 V and s = 5 to ET, Isat = 0.1 A, Vsat = 0.1 V and s = 50 to EER, and the circuit components are set to C1 = 10 pF, C2 = 1 μF, R1 = 1 kΩ, RL = 50 Ω, and Vs is a two-tone independent voltage sinusoidal source to which small and large signals are tested.

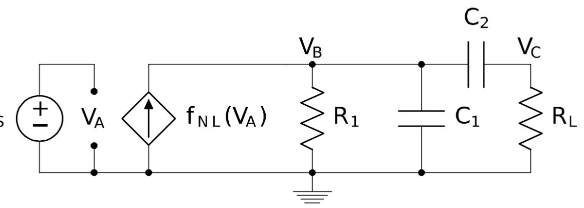

Fig. 1. Circuit schematic adopted in this work.

III. SMALL SIGNAL ANALYSIS

When a nonlinear circuit is stimulated with a sinusoidal source with a lower amplitude, the small signal analysis is validated, and the circuit response can be obtained by DC and AC analyzes associated. To perform small signal analysis, therefore, the circuit is linearized around the DC response and the superposition of the two excitations is applied in the AC analysis.

The small signal analysis response for two-tone circuits is presented by (2) where X_A is the contribution of DC analysis and $x_a(t)$ the contribution of AC analysis.

$$x_A(t) = X_A + x_a(t) = X_A + X_{a1}\sin(\omega_{C1}t + \theta_{a1}) + X_{a2}\sin(\omega_{C2}t + \theta_{a2}) \quad (2)$$

In the DC analysis, the transient is neglected, considering only the steady state, assuming that all voltage and current variations in the circuit are equal to zero, that is, it is assumed that the independent sources of the circuit are constant in time. Thus, the dynamic elements present in the circuit have their characteristics and their behavior altered in the analysis, as observed in the capacitors, which are now considered as open circuits, since there is no voltage variation and, therefore, there is no conduction of current through the element.

The AC analysis is a small signal analysis [6] where, through the phasor representation of voltages and currents, it is possible to analyze the circuit in question. It is necessary that this circuit is linear and all existing independent sources are alternately stimulated at single frequency. Thus, in the AC analysis, all the nonlinear elements are linearized and the behavior of the circuit bipoles are influenced by the alternating source excitation frequency. The impedance resource is used with visible effect on the dynamic elements as capacitors and inductors, and the response of the circuit can be obtained through linear algebraic solution methods.

IV. LARGE SIGNAL ANALYSIS

With the increase of the sinusoidal source amplitudes, there is the appearance of the harmonic components in multiple frequencies of the fundamental frequency of excitation, and the methods of analysis turn out to be large signals, to better approximate the results. In the two-tone power supply in large signals, the phenomenon of the intermodulation product in the frequency domain is identified, in which, in addition to the infinite harmonic components for each of the frequencies that are exciting the circuit, there are also infinite components generated by the combination of the integer multiples of the two excitation frequencies. Therefore, it is necessary to use numerical methods of analysis for large signals such as transient or Harmonic Balance.

A. Transient analysis

The transient response varies at each instant of time with unknowns that depend on previous values, that is, in this analysis there is the dependence with the initial conditions of the circuit. Thus, for its solution, a series of iterations is obtained aiming to analyze the behavior of the component with the time.

In order to develop this type of analysis, the time from initial instant values, sampling interval and final instant (which can also be regarded as the total number of instants of elapsed times) is discretized. In addition, numerical integration methods [6] (such as the Euler method or trapezoidal integration) are used to replace the derivatives of the dynamic elements by algebraic equations.

When there are capacitors in the circuit, its electric current, represented by equation (3), can be discretized in time by the trapezoidal numerical integration method, where ti is the current time and $t_{(i-1)}$ is the instant before the current. For the first iteration of the analysis, it is necessary to use the voltage values of the DC analysis and an initial current value in the capacitors equal to zero, since in DC, the capacitor element can be considered an open circuit since there is no voltage variation in the circuit.

$$ic(ti) = \frac{2 * C}{\Delta t} * [\,Vc(ti) - Vc(t(i-1))] - ic(t(i-1)) \quad (3)$$

B. Harmonic Balance

The transient analysis is used in the simulation of electronic circuits in a nonlinear regime, however, for high frequencies, the quantity of calculations and iterations increases significantly, generating high computational consumption [1], therefore, other methods of analysis are used for this kind of stimulus. Harmonic Balance (HB) consists of the solution of electrical circuits in large signals, whose main objective is to transform all the equations in the time domain into a set of equations independent of each other in the frequency domain.

So, the beat frequency of the circuit is discovered, which is the highest frequency where the two tones are harmonic and, from the beat frequency, the analysis is performed as if it is a one-tone simulation, being possible to use the equation shown in (4). However, the alternative time-frequency transform is done through a procedure similar to the Fourier transform, except that the sum of sines and cosines is also influenced by the products of intermodulation, that is, by the sums and subtractions between the harmonics of the two excitation tones of the circuit. There is also the creation of the Jacobian matrix for the dynamic elements as capacitors, since the derivative found in its characteristic equation must be transformed into the frequency domain.

$$x_A(t) = X_A + \sum_{h=1}^{H} X_{ash}\sin(h\omega_C t) + X_{ach}\cos(h\omega_C t) \quad (4)$$

C. Artificial Frequency Mapping

For the case of two-tone circuits, the solutions are found from a mixing frequency ωk, where ωk is not necessarily a harmonic of a single excitation frequency [2]. The harmonics of the two excitation tones are then replaced by ωk and the voltage, charge and current components are no longer harmonic components, but as a function of ωk. Thus, they must be determined by an alternative time-frequency transform, since the classical Fourier transform is no longer possible to use.

However, to implement the HB with the beat frequency, the more harmonics it is desired to consider in the simulation, the process becomes impracticable due to the large requirement in the multidimensional time-frequency transform. In addition, the frequencies considered in the time-frequency transform with the intermodulation product are not equally spaced, not matching with the sampling performed equally spaced in time, generating inaccurate results. Thus, the method Artificial Frequency Mapping is used, since it seeks to work around this problem, mapping the excitation frequencies so that the intermodulation products are equally spaced [2].

Consequently, we work with a one-dimensional time-frequency transformation matrix. The transform is explained in [2]: for a set of frequencies as expressed in (5), where $0 \leqslant m \leqslant M$ and $|n| \leqslant N$, where $m \neq 0$ when $n < 0$, the coefficients s1 = 1 and s2 given by equation (6):

$$\omega = m\omega 1 + n\omega 2 \tag{5}$$

$$s2 = \frac{\omega 1}{\omega 2 * (2N+1)} \tag{6}$$

are multiplied respectively by mω1 and nω2, obtaining the uniform set of frequencies mapped artificially, possible to adopt the commonly used Fourier transform.

D. *Periodic AC Analysis*

For a two-tone excitation circuit, this method is summarized in the use of the superposition, by applying the harmonic balance twice. It consists of the large signal analysis for the fundamental frequency of the source with greater amplitude of excitation and small signals for the fundamental frequency of the other source, that is, there are components in frequencies $\omega_2 + n\omega_1$ with n ranging from ± 0,1,2,3 until the truncation of the amount of harmonics [2].

Starting from the source with the highest amplitude of excitation and maintaining the characteristics of the equations of linear and nonlinear components, the first HB application is performed. Then, the expression of the capacitor is adapted according to the frequencies of the voltage sources and the linearization of the non-linear component equation is performed based on the data obtained by the first analysis. Thus, the second HB application is performed and the answer will be the sum of the two results.

Considering the contributions of ω_1 and ω_2, the generalized expression of the capacitor currents is presented in equation (7).

$$\begin{aligned} IC = C \cdot [\omega_2 \cdot (v_0 \cdot cos(\omega_2 \cdot t) - v_1 \cdot sen(\omega_2 \cdot t)) \\ +(\omega_2 + \omega_1) \cdot (v_2 \cdot cos[(\omega_2 + \omega_1) \cdot t] - v_3 \cdot sen[(\omega_2 + \omega_1) \cdot t]) \\ +(\omega_2 - \omega_1) \cdot (v_4 \cdot cos[(\omega_2 - \omega_1) \cdot t] - v_5 \cdot sen[(\omega_2 - \omega_1) \cdot t]) \cdots \\ +(\omega_2 + n\omega_1) \cdot (v_6 \cdot cos[(\omega 2 + n\omega 1) \cdot t] - v_7 \cdot sen[(\omega_2 + n\omega_1) \cdot t]) \\ +(\omega_2 - n\omega_1) \cdot (v_8 \cdot cos[(\omega_2 - n\omega_1) \cdot t] - v_9 \cdot sen[(\omega_2 - n\omega_1) \cdot t])] \end{aligned} \tag{7}$$

The linearization of the nonlinear component is performed by applying derivative to the nonlinear equation as a function of the base value for linearization. In (8), G is a vector of conductance amplitudes with k positions obtained with the data from the large signal analysis [2].

$$G = \frac{\partial f(V)}{\partial V}|_{VA} \tag{8}$$

The nonlinear current source acts analogously to a resistance, so it is possible to manipulate the amplitudes of G and to assemble a matrix of conductances by IFL = Cond * V [2]. In this case, IFL is the linearized current that acts where previously there was the nonlinear current source, Cond is the conductance matrix and V is the vector of voltage amplitudes corresponding to the voltage source of the small signal analysis. Hence, we obtain the one shown in (9).

$$IFL = \begin{bmatrix} g_0 & 0 & \frac{g_2}{2} & \frac{-g_1}{2} & \frac{g_4}{2} & \frac{-g_3}{2} & \cdots & \frac{-g_{k-2}}{2} \\ 0 & g_0 & \frac{g_1}{2} & \frac{g_2}{2} & \frac{g_3}{2} & \frac{g_4}{2} & \cdots & \frac{g_{k-1}}{2} \\ \frac{g_2}{2} & \frac{g_1}{2} & g_0 & 0 & \frac{g_2}{2} & \frac{-g_1}{2} & \cdots & \frac{-g_{k-4}}{2} \\ \frac{-g_1}{2} & \frac{g_2}{2} & 0 & g_0 & \frac{g_1}{2} & \frac{g_2}{2} & \cdots & \frac{g_{k-3}}{2} \\ \frac{g_4}{2} & \frac{g_3}{2} & \frac{g_2}{2} & \frac{g_1}{2} & g_0 & 0 & \cdots & \frac{-g_{k-6}}{2} \\ \frac{-g_3}{2} & \frac{g_4}{2} & \frac{-g_1}{2} & \frac{g_2}{2} & 0 & g_0 & \cdots & \frac{g_{k-7}}{2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ \frac{-g_{k-2}}{2} & \frac{g_{k-1}}{2} & \frac{-g_{k-4}}{2} & \frac{g_{k-3}}{2} & \frac{-g_{k-6}}{2} & \frac{g_{k-7}}{2} & 0 & g_0 \end{bmatrix} \cdot \begin{bmatrix} v_{k-1} \\ \vdots \\ v_4 \\ v_5 \\ v_0 \\ v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_{k-2} \end{bmatrix} \tag{9}$$

V. SIMULATION RESULTS

The following section presents the results obtained in simulations using Matlab software as a mathematical library for calculation and repetition of iterations of the methods.

First of all, the small signal analysis was compared to the transient for amplitudes A1 and A2 of the voltage source presented in equation (10), respectively, equal to 0.01 V and 0.05 V using the ERR topology. Under this condition, it was possible to observe and verify the operation of both analyzes for small signals, as shown by the overlapped curves in Fig. 2.

$$Vs = A1 * \sin(\omega 1t) + A2 * \sin(\omega 2t) \tag{10}$$

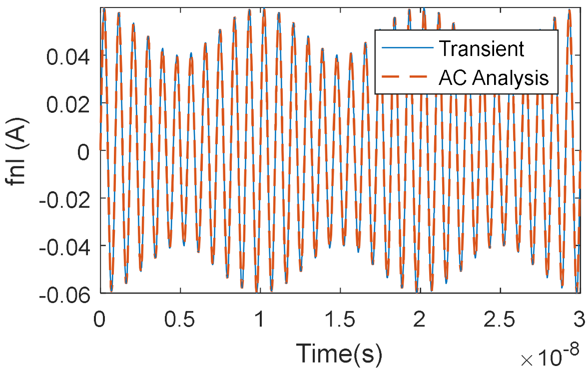

Fig. 2. Small Signal Analysis and Transient Response Comparison for Lower Amplitudes.

With the increase of amplitudes for A1 = 1.8 V and A2 = 0.9 V in the ET topology, the analysis of small signals does not converge to the expected result. Thus, looking at Fig. 3, we mean the need to employ one large signals analysis.

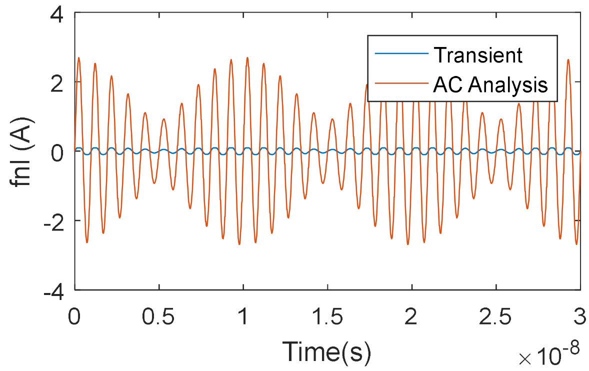

Fig. 3. Small Signal Analysis and Transient Response Comparison for Larger Amplitudes.

Hence, the PAC analysis was used, the equation (11) was used for the formation of the matrix of conductance, with the ET parameters and the results are presented in Fig. 4.

$$G = \left[\frac{2 \cdot Isat \cdot dirac(VA)}{\left(\left(\frac{Vsat}{|VA|}\right)^{s}+1\right)^{\frac{1}{s}}} + \frac{\left((Isat \cdot Vsat \cdot sign(VA))^{2} \cdot \left(\frac{Vsat}{|VA|}\right)^{(s-1)}\right)}{|VA|^{2} \cdot \left(\left(\frac{Vsat}{|VA|}\right)^{s}+1\right)^{\frac{1}{s}+1}} \right] \quad (11)$$

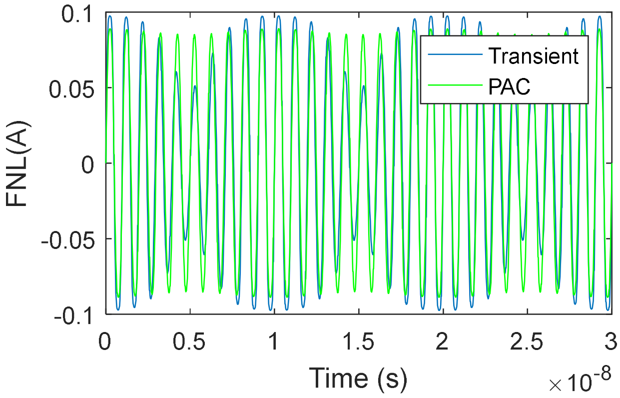

Fig. 4. Periodic AC Analysis and Transient Response Comparison for Larger Amplitudes.

Since the nonlinear source was linearized for the application of the small signal analysis, the response signal never saturates in Isat, consequently for large signals added in the second application of the HB, the response signal exceeds the Isat. Therefore, since the output exceeds the saturation current of the non-linear source, it is possible to attest that the PAC does not work for large signals.

It was also compared the artificial frequency mapping analysis with the transient, a method that was verified by the one presented in Fig. 5.

Therefore, we can observe that the method of artificial frequency mapping is valid for both conditions, as shown in Fig. 6, where results are presented for the topology ERR, with small signals. Hence, among the four methods studied, this one is presented as the most efficient, since it met the two desired cases and requires less computational processing.

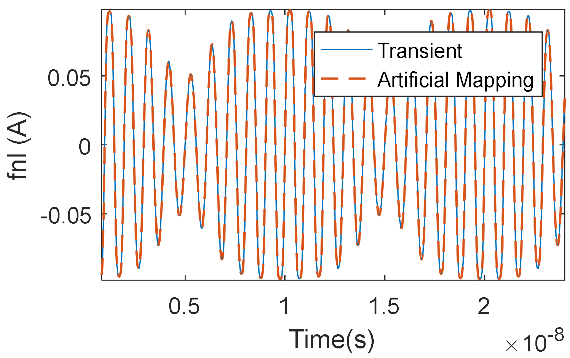

Fig. 5. Artificial Frequency Mapping and Transient Response Comparison for Larger Amplitudes.

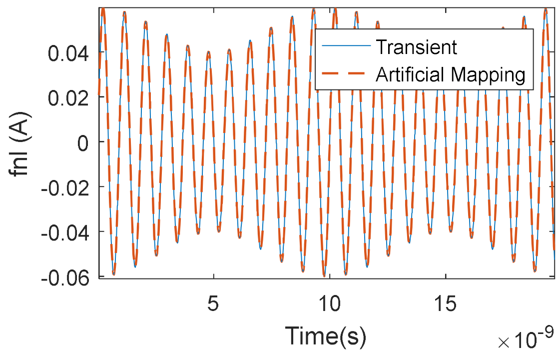

Fig. 6. Artificial Frequency Mapping and Transient Response Comparison for Lower Amplitudes.

VI. Conclusions

The aim of this work was to present and compare two-tone circuit analysis methods, adopting four main methods: small signal analysis, transient analysis, periodic AC analysis and artificial frequency mapping. It can be observed that the nonlinearity affects the circuits excited to large signals. Even though they are accurate, transient analysis lead to higher processing costs or mathematical calculations. For the configurations of the PA used in the tests (ET and EER), the Artificial Frequency Mapping shows the most efficient method.

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