Behavioural modelling with the reception of a modulated signal of a radio-frequency receiver based on N-path filters and mixers

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***Abstract*—Over the past decades, the advances in wireless technology and semiconductor devices have pushed the development of multi-band reconfigurable wireless transceivers. To avoid bulky off-chip filters, the concept of N-path filters and mixers is one of the most promising techniques to be explored. In this work, a theoretical analysis is presented to model an entire N-path based receiver. A 16-QAM modulated signal is applied at the input of the 4-path mixer-first based receiver. System-level simulation is provided through MATLAB - Simulink tools. As output metrics, the modulated signal constellation diagram and symbol-error-rate (SER) are presented, confirming the successful demodulation of the signal. At the baseband, there was approximately 45° phase shift, indicating the system's delay, which can be explained by looking at the switched-RC kernels harmonic transfer functions.**

***Index Terms*—N-path filters, switched R-C circuits, RF receivers**

I. Introduction

With the improvement of wireless communication technologies and a frequency spectrum increasingly congested, it is necessary to use high selective filters, reconfigurable by software, and whose central frequency is variable. In recent years, due to semiconductor devices' current capabilities, a discussion around the N-path mixer-first receivers and filters with high switching frequencies and high selectivity has been formed and consolidated [1].

An N-path filter or mixer is composed of N identical switched R-C kernels (or meshes), switched one after the other between the system's input and output during a switching period. An R-C circuit's low-pass transfer function is frequency shifted to the radio frequencies in an N-path filter. The more R-C kernels compose the filter, the higher is the out-of-band rejection [1]. Furthermore, by applying differential circuits, one can design an N-path mixer-first receiver and filter, where it is possible not only to filter but also, to downconvert the received signal to baseband [2]. With this structure, separating the in-phase and quadrature components of the information transmitted from a transmitter to a receiver is possible. A theoretical analysis for the N-path filters and mixers is presented in this paper. After that, a communication system simulation using these filters and mixers as receivers will be discussed, and its results provided.

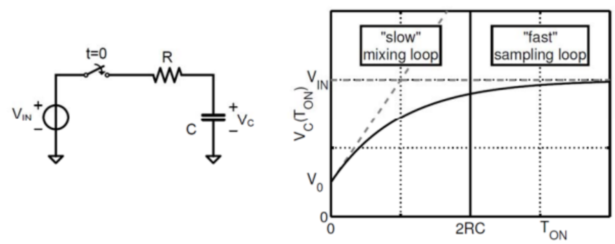


Fig. 1: Switched R-C circuit and its respective time behaviour [1].

II. N-Path Filters and Mixers

A. Switched R-C Circuits

A switched R-C circuit consists of a resistor, a switch and a capacitor, as shown in Fig. 1 [1]. A switched R-C circuit has two different operating modes. The more classical application is the sample-and-hold circuit. The less explored mode is the mixer. The operating mode depends on the relation between the time the switch remains closed, T_{ON}, and the time constant $\tau = RC$ of the R-C circuit, as shown in [1].

The time interval in which the switch remains closed is defined as follows:

$$T_{ON} = DT, \tag{1}$$

where $T = 1/f_s$ is the switching period of the switched R-C circuit (f_s being the sampling frequency), and D is the duty cycle of the circuit. Another parameter to be defined is Γ [1]:

$$\Gamma = \frac{T_{ON}}{\tau} = \frac{DT}{RC}. \tag{2}$$

From Fig. 1, one can observe that if $T_{ON} >> 2RC$ or, equivalently, if $\Gamma >> 2$, the circuit will be operating in the "fast" sampling loop. Otherwise, it will be operating in the

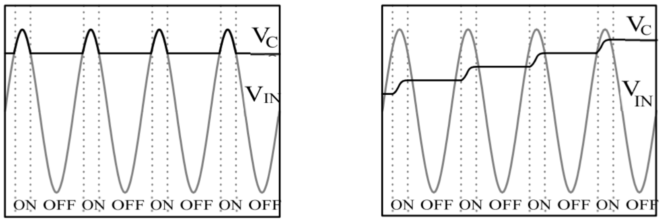


Fig. 2: Time behaviour of a switched R-C circuit for (a) a fast loop with $\Gamma >> 2$ and (b) a slow loop with $\Gamma << 2$ [1].

"slow" loop, that is, as a mixer [1]. This behavior is captured in Fig. 2.

This work focuses on the behavior of the switched R-C circuit acting as a mixer, that is, in $\Gamma << 2$. In this mode of operation, it is possible to build an N-path filter.

Using the concepts of linear periodically time-varying systems, as seen in [3], the switched R-C circuit has harmonic transfer functions which are dependent on Γ. In the case where $\Gamma << 2$, the harmonic transfer functions are demonstrated by [1] as

$$H_k(f - kf_s) \approx \frac{\mathrm{sinc}(D \cdot k)}{1 + \frac{j(f-kf_s)}{D \cdot f_{RC}}} e^{-j\pi Dk} \tag{3}$$

where $f_{RC} = \frac{1}{2\pi\tau} = \frac{1}{2\pi RC}$ and k is the harmonic index of the corresponding switching frequency of the switched R-C kernel, f_s.

From (3), the harmonic transfer functions of a switched R-C circuit are composed of a low pass filter with a bandwidth equal to the circuit's duty cycle D multiplied by its cutoff frequency f_{RC}, represented by the factor $D \cdot f_{RC}$. The amplitude gain and phase of the harmonic k at the output are given, respectively, by the factors $\mathrm{sinc}(Dk)$ and $e^{-j\pi Dk}$ [1].

B. N-Path Filters and Mixer-First Receivers

In an N-path filter, each one of the N kernels remains connected with the source for a period $T_{ON} = \frac{1}{N} \cdot T$ while the other $N-1$ kernels are, ideally, disconnected from it. One can represent the switches' resistance, the R-C circuit resistance itself, and the source resistance as a single resistor between the source at the input and the node from which the N-path filter output is obtained [2]. Thus, the topology of a single-ended N-path filter is seen in Fig. 3.

In an N-path filter, the switching frequency f_s and the duty cycle D are the same for each of the N switched R-C circuits that make up the system kernels. Since these N switched kernels are connected to the input source one at a time in a single switching period T of the filter, the duty cycle of the kernels must be given by the relationship (4)

$$D = \frac{1}{N} \tag{4}$$

A switched R-C circuit has harmonic transfer functions given by (3). Also, in a N-path filter, each of the R-C circuits

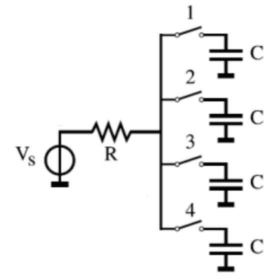


Fig. 3: 4-path single-ended N-path filter with 25% duty cycle [2].

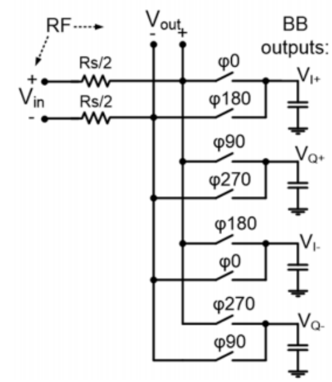


Fig. 4: 4-path mixer-first receiver with differential input and output and a duty cycle of 25% [2].

is switched with a switching period $T = \frac{1}{f_s}$ and a duty cycle $D = \frac{1}{N}$, as shown in (4). The delay between the kernels is $\frac{1}{N} \cdot T$ Consequently, the filter's output voltage, V_{out} is the sum of the voltages on the capacitors of the RC kernels, multiplied by their respective square waves that represents the switching of each of these kernels, delayed by $\frac{1}{N} \cdot T$ from one another. Since a multiplication in the time domain is equivalent to a convolution in the frequency domain, we have that the total transfer function of the N-path filter, from the input $V_{in}(f)$ to the output $V_{out}(f)$ is given by (5) [1]

$$H_{in}(f) = \sum_{i=0}^{N-1} \Bigg[e^{-\frac{j2\pi \cdot i}{N}} \cdot \sum_k (\mathrm{sinc}(Dk) \cdot H_{n,SE}(f - kf_s) \cdot e^{-\frac{j2\pi \cdot i \cdot n}{N}}) \Bigg] \tag{5}$$

Finally, the single-ended 4-path filter in Fig. 3 can be converted into its differential version, as shown in Fig. 4, commonly known as 4-path mixer-first [2]. This particular topology implements downconversion along with the filtering. The output is the complex baseband of a signal at its input with components in-phase and quadrature at the switched R-C kernels output. The mixer-first topology is particularly interesting for software-defined radio systems, whose one of its characteristics is the central frequency controlled by software.

III. METODOLOGY

The developed MATLAB/Simulink model for the N-PATH / Mixer-First receiver is illustrated in Fig. 5. With this model, it is possible to trace and evaluate the receiver's behavior in the demodulation of a radiofrequency signal.

The encoded symbols (generated at a rate of $R_{sym} = 10$ Mbaud) are filtered by a root-raised-cosine pulse shaping filter (FIR) with an oversampling rate of 3200 samples per symbol. Such high simulation sampling frequency is required due to the switching nature of the circuit, which is simulated at the RF frequency.

Then, the pulses from the in-phase signal path are mixed with a cosine with amplitude 2V and frequency 1 GHz, and the pulses from the quadrature signal path are mixed with a 2V amplitude sine wave of that same frequency. After that, the signals from each of the paths are then subtracted from one another which is equivalent to transmitting only the real part of the signal's complex representation [4]. Finally, the signal that results from this subtraction is transmitted through an ideal channel. This is the signal that must be received and demodulated by the receiver in question.

This same procedure was then repeated for a different 16-QAM signal to be transmitted through the same physical channel as the first one, but with a different carrier frequency. This new 16-QAM signal represents a possible in-band or out-of-band blocker in the channel. However, for the purposes of this study, this new 16-QAM signal was not taken into account in the simulation and will not be discussed any further.

At the reception, using a 4-path mixer-first receiver with differential input and output, as seen in Fig. 4, both passband filtering (in radiofrequency) and down-conversion back to baseband of the received signal occurs. The received baseband signals, both in-phase and in quadrature, are obtained through the outputs Vi _P, Vi _N, Vq _P, and Vq _N of the 4-path mixer-first receiver. These are the voltages on the capacitors of each of the four switched kernels of the 4-path mixer-first. This is possible due to the N-path filters and mixers' properties of filtering and down-conversion back to the baseband. After that, the two baseband signals are then filtered using a root-raised-cosine pulse shaping filter matched with that same filter at the transmitter side. Now, knowing that a 4-path mixer-first receiver phase shifts the signal by 45 degrees, a complex phase shift compensation of 45 degrees is applied on the received signal. This signal is then decimated, and a 16-QAM constellation diagram is built at the reception. The received in-phase and quadrature signals are then quantized and decoded (using Gray code), and the computational delay between the transmitted and received 16-QAM signals is estimated. Finally, after aligning the received and transmitted 16-QAM signals by delaying the latter using the amount of samples previously estimated, it's possible to calculate the symbol-error-rate achieved by the system when comparing them both.

IV. RESULTS

As the main focus of this paper is to understand and start modeling the behavior of a receiver based on N-path filters and mixers in a communication system, the relevant results of the simulation are those related to the correct (or incorrect) reception of the symbols that were transmitted in the first place. One of these results is the 16-QAM constellation diagram built after the downsampling of the output in-phase and quadrature baseband signals from the 4-path mixer-first receiver.

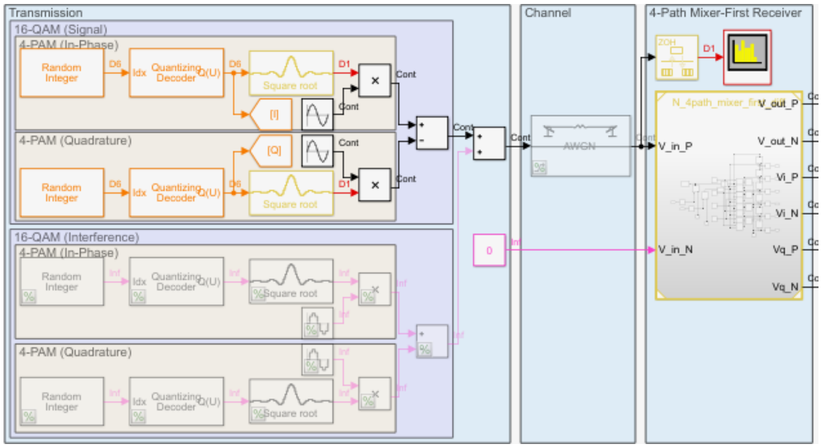
In Fig. 6a, where the constellation diagram of the received in-phase and quadrature symbols can be seen, it is possible to note that a rotation of approximately +45 degrees has occurred in the diagram. This rotation is visible when noting that the diagram has 16 well-defined regions (represented by yellow dots), which is a consequence of the 16 possible QAM symbols that can be received. When comparing these regions' centers positions with the reference 16-QAM constellation (the red '+' indications on the diagram), a 45-degree rotation is seen. This same diagram can be seen in Fig. 6b after applying a complex phase shift correction of -45 degrees on the received 16-QAM signal.

Ideally, the more concentrated around each of the red '+' indications the yellow dots are, the more adequate the reception will have been in terms of symbol-error-rate (SER) and, consequently, bit error rate. This rotation observed in the constellation diagram of Fig. 6a as a whole (which gives it a shape that resembles that of a rhombus, that is, a 45-degree rotated square) indicates the presence of a delay in the in-phase and quadrature signals. This +45-degree delay in the baseband signals, both in-phase and quadrature, can be explained when considering the harmonic transfer functions of a switched R-C kernel by observing (3). Since the baseband signals are obtained at the reception by observing the voltages on the capacitors of the R-C kernels that make up the 4-path mixer-first receiver, and knowing that the carrier frequency of these signals is equal to the switching frequency of the filter, that is, 1 GHz, the harmonic transfer function that must be observed is the one corresponding to the 1st harmonic, or the index k = 1. For k = 1, and knowing that in a 4-path mixer-first receiver the duty cycle D is equal to 25%, we have from (3)

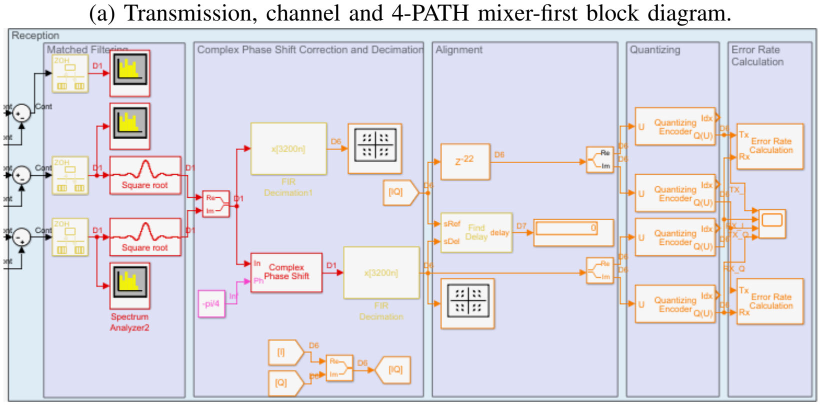
$$H_1(f - kf_s) = \frac{\mathrm{sinc}(0.25)}{1 + \frac{j(f-kf_s)}{0.25 f_{RC}}} e^{-j\frac{\pi}{4}}$$

in which it is possible to see from the term $e^{-j\frac{\pi}{4}}$ that a phase shift of $-\frac{\pi}{4}$ (or -45 degrees) occurs at the reception. However, because the received signal's quadrature component (or imaginary component) is phase shifted by 180 degrees in an image-reject down-converter mixer [4], this -45 degrees rotation becomes a +45 degrees rotation which, finally, reflects the constellation diagram of the Fig. 6a, and that is further compensated by applying a -45 degrees phase shift to the 16-QAM signal at reception.

Another important result is the symbol-error-rate (SER) achieved by the system as a whole. Because non-idealities are not taken into account in this first study, e.g., noise, interference and others, it is expected for the SER of the system to be small or even 0. For a simulation of 699 16-QAM

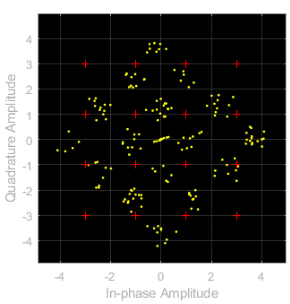


(a) Transmission, channel and 4-PATH mixer-first block diagram.

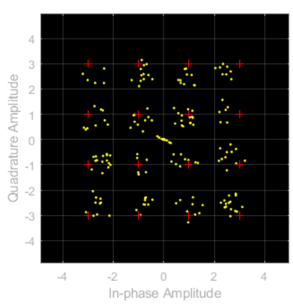


(b) Blocks diagram at reception.

Fig. 5: Simulink's block diagram of a simulation of a wireless communication system using a 4-path mixer-first receiver.



(a) Before the -45º phase shift.



(b) After the -45º phase shift.

Fig. 6: Received signal's constellation diagram (a) before and (b) after the complex -45 degrees phase shift.

symbols, the symbol-error-rate (SER) indeed results 0% which shows that the 4-path mixer-first based receiver successfully demodulated the signal at reception.

V. Conclusion

This work presented the simulation of an ideally modeled wireless communication system using an N-PATH mixer-first based receiver without channel noise or in-band or out-of-band blockers. At the output, one can observe that the differences between the expected 16-QAM constellation diagram and the output constellation diagram are restricted solely to the N-path filters and mixers and their switched R-C kernels intrinsic characteristics. The following steps of this study of receivers based on N-path filters and mixers will take place through the addition, first, of noise and, later, of in-band and out-of-band blocker signals in the communication channel from the transmission to the reception, showing the interferers rejection through the N-PATH mixer-first receiver.

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