Circuit Analysis Based on Linearization Around Circuit Envelope

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Abstract — The objective of this article is to introduce and show the validity limits of a proposed method for the analysis of circuits, called Combined Non-linear and Linearized Circuit Envelope (CNL-CE). For that it was necessary to understand the functionality of some methods of circuit analysis, such as Transient analysis, Harmonic Balance (HB), Periodic Alternating Current (PAC) and Circuit Envelope (CE), in addition to the methods of linearization and superposition of circuits. The objective of the CNL-CE is to perform the linearization of circuits around the CE analysis, similarly to the PAC method around HB analysis. Considerations about these analyses are based on comparisons made in the Matlab environment, concluding with discussions on the effectiveness of the proposed method compared to the transient analysis.

Keywords — Linearization, Circuit Envelope, Simulation

I. INTRODUCTION

The use of simulations on the area of electrical and electronic circuits is recommended for all types of circuits arrangements, from the simplest to the most complex ones. Circuit simulations offer only advantages to the designers. Through them it is possible to verify if the circuit behaves as expected, there is high flexibility regarding error correction and circuit optimization, as well as preventing the implementation of circuits with mistakes, therefore helping in saving resources as time, materials and manpower. The simulation of radio frequency circuits is a subject that is currently of high interest [1] and [2].

There are several circuit simulation environments, differentiated by the method of analysis used, which depend on the time progress of the independent sources and of the need, or not, of the part regarding to the transient response [3]. Referring to the analysis of circuits in large signals, which has as circuit power, independent sources of voltage and/or current periodic over time acting in “x” tones, the steady state is the range of interest. This analysis can be approached both in time and frequency domain. When the analysis occurs in the frequency domain, by applying the harmonic balance (HB) method, the unknowns are amplitudes, constants over time, of sine and cosine, and after acquiring those amplitudes it is possible to get all waveforms of voltage or current for any instant in time [3]. In the time domain, by applying the transient analysis, the unknowns are values of voltage or current for instants equally spaced in time, that is, unlike the analysis in frequency domain, the values of voltage and current are available only for a finite amount of instants in time [3].

Considering the particularities described above regarding the analysis in frequency and time domain, it is possible to say that in wireless communication systems, powered by digitally modulated signals, where the bandwidth is much lower than the central frequency and most part of the spectrum is null, the transient analysis would be applicable, however it would require high computational effort. On the other hand, the harmonic balance method would not be applicable, because the HB method requires that the power source acts only periodically in time. Another alternative to perform the analysis on a circuit with the specific characteristics described before would be the Circuit Envelope (CE) method, which combines both transient and HB analysis, and because of that it is able to realize calculus only for the non-null spectral components.

In the presence of two or more carriers the CE method treats all carriers in a non-linear way. This research paper proposes one method which linearizes the circuit around the response acquired on the application of the circuit envelope method in the presence of the highest power carrier. The carriers of lower amplitudes are taken into consideration on the linearized circuit. The superposition analysis is applied to obtain the complete circuit response.

II. CIRCUIT ANALYSIS METHODS

A. Harmonic Balance (HB)

The HB method works for both low and high input signals and operates on a steady state basis. This method obtains, from a modified nodal analysis, equations independent of each other and carries out the transformation of all these equations from the time domain to the frequency domain. The equations are represented by a group of (2N+1), where N represents the amount of considered harmonics, amplitudes of sines and cosines in addition to the component of direct current, and with only one solution of the non-linear algebraic system all the constant values of the unknowns are obtained, allowing the representation of any voltage or current of the analyzed circuit at any instant of time.

For the correct application of this method, all the independent sources of the analyzed circuit must be periodic in time.

B. Transient Analysis

The transient analysis occurs in time domain and can be used to obtain the complete response of circuits. In this kind of analysis, the values of initial instant, time step and final instant must be provided by the user. The time vector must be equally divided over time, in order to respect the Nyquist criterion ($F_s > 2*f_{máx}$), which says that the sampling frequency must be larger than two times the highest frequency of the sampled signal. To replace each differential equation of the dynamic elements, a numerical integration method must be applied, such as Euler or Trapezoidal.

Each iteration of this method depends on the previous solution, due to the consideration of the memory of the dynamic elements present in the circuit. The complete response of this analysis consists on a series of solutions of the non-linear algebraic system for each discretized time instant, from initial instant to final instant.

C. Circuit Envelope (CE)

The CE method is an applicable circuit analysis for circuits where the amplitudes of sine and cosines vary in time according to:

$$x(t) = X_0(t) + \sum_{n=1}^{N} X_{ns}(t) sin(n\omega_1 t) + X_{nc}(\mathrm{t}) \cos(n\omega_1 t) \quad (1)$$

In (1), $X_0(t)$, $X_{ns}(t)$ and $X_{nc}(t)$ are amplitudes that vary slowly in time, which means that in frequency domain the bands do not overlap, that is, the spectral content of each time varying amplitude cover a tiny space around the frequency of the sines and cosines, forming a continuous band, as shown in Fig. 1.

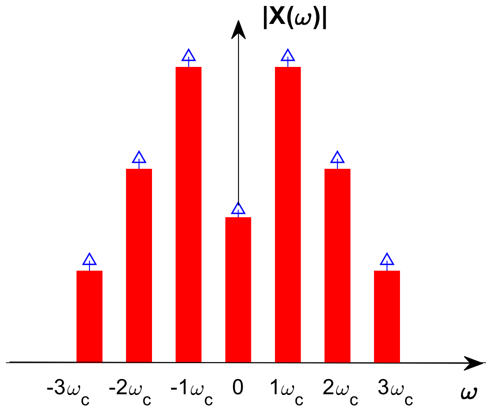

Figure 1 – Representation of the amplitude spectrum for the CE analysis

In Fig. 1 the blue Dirac deltas represent the periodic signals of the HB method and the red ones represent the signals of the CE method.

The CE method combines the functionalities of the transient and HB analysis, by applying a modified HB in each time instant of the transient analysis. In this analysis the original equations of the HB are maintained. The adaptation happens only in the equations of the dynamic elements, where at each time step the analysis considers the memory of the dynamic elements and solves the non-linear algebraic system, obtaining, in each discretized time, amplitudes of sines and cosines. In the end of the simulation the response is composed of a set of amplitudes of sines and cosines that vary in time. In the CE method, the carrier is treated in frequency domain, where its values of phase and amplitude are applied on the portion of the analysis referring to HB and the envelope is treated in time domain, because the envelope normally varies more slowly than the carries of the signal.

This method is only suitable for circuits that have digitally modulated inputs and can be applied in situations where there is more than one carrier, as long as the carriers are multiple of each other. Among its restrictions, there is also the need of the independent sources to stimulate only frequencies contained within bands equally spaced in frequency. The spacing between the different bands must be much larger than the width of these bands.

In circuits which attend those specifications, the CE method has advantage over both HB and Transient analysis. In the CE method the Nyquist criterion is dictated only by the envelope. Due to this, the time step used is much larger and, therefore, the amount of calculus required is much less compared to a transient analysis.

The CE came to adapt the HB method for analyses of circuits such as the described before. In this method the calculus is made only where the spectrum is non-null, therefore a transient analysis is applied using as $f_{máx}$ the envelope bandwidth and the calculus are carry out for $(2N + 1)$ bands.

III. Proposed Method – Combined Non-linear and Linearized Circuit Envelope (CNL-Envelope)

In the presence of two or more carriers, the CE method treats all carriers in a non-linear way. The proposed method in this work treats in a non-linear way only the carrier of the greatest amplitude, analyzing the others in the linear circuit.

A. Linearization and Superposition

In a general way, the procedures for the linearization of a non-linear equation remains the same, even for different non-linear equations and different non-linear components. Basically, the first order of Taylor series is applied in the non-linear equation as a function of its initial trajectory:

$$f_{NL}(x) = \frac{df_{NL}(x)}{dx}|_{x=x_0} \cdot (x - x_0) \quad (2)$$

The superposition method performs the analysis of a circuit with two or more independent power sources adding up the influence of each input acting alone in the circuit. The final complete equation of a circuit that suffers the linearization is given by:

$$x_{total}(\mathrm{t}) = x_0(t) + x_{new}(\mathrm{t}), \quad (3)$$

where $x_0(t)$ and $x_{new}(\mathrm{t})$ refer to the non-linear and linearized response of the circuit, respectively. That means, $x_{new}(\mathrm{t})$ is the share of the circuit that suffers the linearization in function of $x_0(t)$.

B. Linearization around Circuit Envelope Analysis

The linearization around CE analysis, as the CE method, combines the functionalities of analysis both in time and frequency domain. This analysis considers the memory of dynamic elements and it is based on trigonometric deductions, which depend on the amount of harmonics to be considered in the analysis.

This linearization works similarly to the method known in the literature as Periodic Alternating Current (PAC). In the CNL-CE, one original analysis of CE is performed on the isolated action of the carrier with the highest power, composing the $x_0(t)$ response of the superposition analysis. Subsequently the linearization of the non-linear component equations occurs and then one linear analysis of small signals is performed in the action of the carriers of lower power, composing the $x_{new}(\mathrm{t})$ response of the superposition analysis. The full circuit response $x_{total}(\mathrm{t})$ is exemplified in (3).

The initial trajectory used in the linearization of non-linear components is the response to large signal analysis, $x_0(t)$. Therefore, performing the linearization procedures and rearranging some components it is possible to obtain the equation:

$$x_{1new}(\mathrm{t}) = g_0(\mathrm{t}) \cdot x_{\mathrm{new}}(t) \tag{4}$$

where $x_{1\mathrm{new}}(\mathrm{t})$ and $x_{\mathrm{new}}(t)$ indicate the linearized parcels around $x_0(t)$, of the unknowns $x_1(t)$ and $x(t)$, respectively. The $g_0(\mathrm{t})$ refers to the derivative of the non-linear component function around the initial trajectory. Since the multiplication $g_0(\mathrm{t}) \cdot x_{\mathrm{new}}(t)$ is basically composed by trigonometric iterations as sine sine, cosine cosine or sine cosine, when applied some trigonometric properties, it is possible to rearrange (4) in a multiplication of matrices, according to:

$$x_{1new}(\mathrm{t}) = \overrightarrow{\mathrm{G(t)}} \cdot \overrightarrow{\mathrm{X}(t)} \tag{5}$$

where $\overrightarrow{\mathrm{G(t)}}$ represents a time varying conductance matrix and $\overrightarrow{\mathrm{X}(t)}$ represents a vector composed of sines and cosines, which is dependent on the amount of considered harmonics.

The conductance matrix $\overrightarrow{\mathrm{G(t)}}$ has its order k determined by $2 * (2N+1)$, where N refers itself to the quantity of harmonics considered in the large signal analysis, according to:

$$\begin{bmatrix} g_0(t) & 0 & \frac{g_2(t)}{2} & \frac{-g_1(t)}{2} & \frac{g_{4}(t)}{2} & \frac{-g_3(t)}{2} & \cdots & \frac{-g_{k-3}(t)}{2} \\ 0 & g_0(t) & \frac{g_1(t)}{2} & \frac{g_2(t)}{2} & \frac{-g_3(t)}{2} & \frac{g_4(t)}{2} & \cdots & \frac{g_{k-2}(t)}{2} \\ \frac{g_2(t)}{2} & \frac{g_1(t)}{2} & g_0(t) & 0 & \frac{g_2(t)}{2} & \frac{-g_1(t)}{2} & \cdots & \frac{-g_{k-5}(t)}{2} \\ \frac{-g_1(t)}{2} & \frac{g_2(t)}{2} & 0 & g_0(t) & \frac{g_1(t)}{2} & \frac{g_2(t)}{2} & \cdots & \frac{g_{k-4}(t)}{2} \\ \frac{g_4(t)}{2} & \frac{g_3(t)}{2} & \frac{g_2(t)}{2} & \frac{g_1(t)}{2} & g_0(t) & 0 & \cdots & \frac{-g_{k-7}(t)}{2} \\ \frac{-g_3(t)}{2} & \frac{g_4(t)}{2} & \frac{-g_1(t)}{2} & \frac{g_2(t)}{2} & 0 & g_0(t) & \cdots & \frac{g_{k-8}(t)}{2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ \frac{-g_{k-3}(t)}{2} & \frac{g_{k-2}(t)}{2} & \frac{-g_{k-5}(t)}{2} & \frac{g_{k-4}(t)}{2} & \frac{-g_{k-7}(t)}{2} & \frac{g_{k-8}(t)}{2} & 0 & g_0(t) \end{bmatrix} \tag{6}$$

where the conductance indices are numbered in ascending order of harmonics. In (6), odd indices indicate amplitudes of sines and even indices indicate amplitudes of cosines. The correspondent vector $\overrightarrow{\mathrm{X}(t)}$ is given by:

$$\begin{bmatrix} x_{k-2}(t) \cdot sen[(\omega_2 - n\omega_1)t] \\ x_{k-1}(t) \cdot cos[(\omega_2 - n\omega_1)t] \\ \vdots \\ x_8(t) \cdot sen[(\omega_2 - 2\omega_1)t] \\ x_9(t) \cdot \cos[(\omega_2 - 2\omega_1)t] \\ x_4(t) \cdot sen(\omega_2 - 1\omega_1)t] \\ x_5(t) \cdot \cos[(\omega_2 - 1\omega_1)t] \\ x_0(t) \cdot sen(\omega_2)t \\ x_1(t) \cdot \cos(\omega_2 t) \\ x_2(t) \cdot sen[(\omega_2 + 1\omega_1)t] \\ x_3(t) \cdot \cos[(\omega_2 + 1\omega_1)t] \\ x_6(t) \cdot sen[(\omega_2 + 2\omega_1)t] \\ x_7(t) \cdot \cos[(\omega_2 + 2\omega_1)t] \\ \vdots \\ x_{k-4}(t) \cdot sen[(\omega_2 + n\omega_1)t] \\ x_{k-3}(t) \cdot cos[(\omega_2 + n_1\omega_1)t] \end{bmatrix} \tag{7}$$

The CNL-CE is indicated for non-linear circuits that have one or more carriers, where the interest lies in the linear response to the small signals.

IV. Simulation Results

For the validation and verification of the limits of application of the CNL-CE method, the test circuit shown in Fig. 2 was used [4].

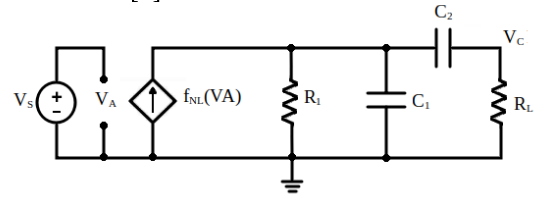

Figure 2 – Schematic of the test circuit

In Fig. 2, Vs represents the power source of the circuit, which shall contain digitally modulated signal as input. The parameter set for this circuit is $\mathrm{C}_1 = 10$ pF, $\mathrm{C}_2 = 1$ µF, R1 = 1 kΩ and $\mathrm{R_L} = 50$ Ω. The non-linear component of the circuit is the voltage-controlled current source whose expression is the same used in [5].

For the validation tests, two digitally modulated inputs were used, where the amplitudes of sines and cosines vary in time. The carrier frequencies of large signal and small signal were fixed in 1 GHz and 2 GHz, respectively. The original signals used for the large (V_{ls}) and small (V_{ss}) signal analysis are shown in Fig. 3.

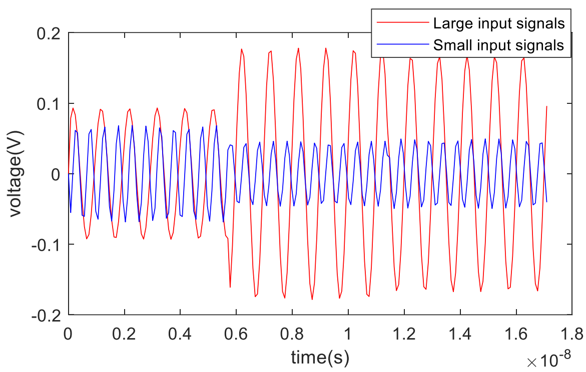

Figure 3 – Waveforms of large and small input signals

The limits of application of the proposed method were analyzed by means of several comparisons in relation to the transient analysis. Knowing that the critical portion of the CNL-CE is in the analysis of small signals, the parameters of the large signal analysis were fixed and only the multiplier of the small input amplitudes is varied at each test.

When multiplying the signal V_{ss} by 15 and plotting the comparison between Transient and CNL-CE analysis, Fig. 4 shows that there is a small variation of amplitudes in the comparison. So small that it is almost not visible in Fig 4. For this case the mean square error (MSE) is $6.92\,x\,10^{-08}$, therefore, it was concluded that the method is valid for the used parameters.

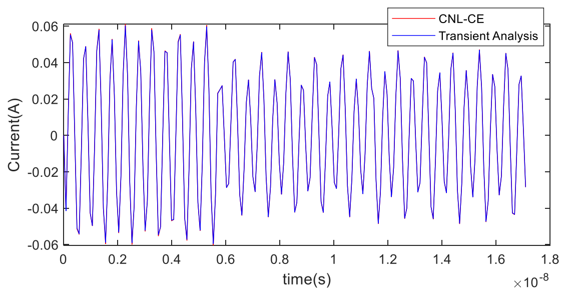

Figure 4 – Waveforms for small input signals with multiplier equal to 15

When multiplying the signal V_{ps} by 30 and plotting the comparison between Transient and CNL-CE analysis, Fig. 5 shows that the variation between the two curves is far more significant than the one shown in Fig. 4. This happens due to the linearization process. The MSE for this case is $4.80\, x\, 10^{-05}$, which is a lot bigger than the MSE for the previous case. It was concluded that the proposed method is not valid for the parameters used in this simulation.

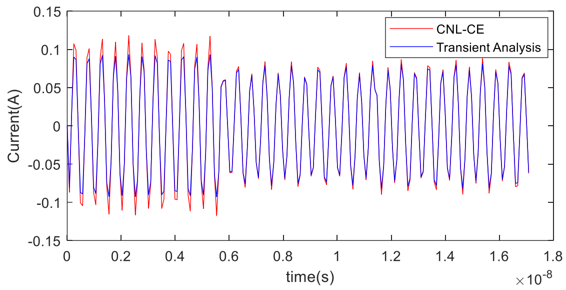

Figure 5 – Waveforms for small input signals with multiplier equal to 30

V. Conclusion

The objective of this research was to develop, test, validate and explore the validity limits of a method based on Periodic Alternating Current, which we call Combined Non-linear and Linearized Circuit Envelope. Then, linearizing the circuit around the CE analysis it was possible to conclude that the proposed method works, but, like any other method, it has limitations. From the studies carried out in this research work, it was understood that the input that suffers the linearization is limited to receiving only signals with amplitudes in small signals, while the input that integrates the linearization can act with amplitudes in both small and large signals.

As future works, it is intended to apply the method developed in this research in the analysis of digitally pre-distorted power amplifiers.

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