

# Behavioral Models of Power Amplifier Using Multiple One-Dimensional Polynomial Functions and Multiple Finite Impulse Response Filters

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**Abstract**—Power amplifiers (PA) are essential in data transmission and communication without wire. For a better operation a linear behavior is important. In this work, it is compared the behavior of four PA models, three from the literature and one proposed model, using a GaN transistor data set. All the models were run in the environment MATLAB. To compare the results, NMSE (Normalized Mean Square Error) was used, and the results were divided according to the value of the memory length  $M$ . For  $M=1$  the literature model with a single filter and multiple polynomial functions (F1P+) produced an NMSE of -36.49 dB, the literature model with multiple filters and multiple polynomial functions (Previous F+P+) shown an NMSE of -36.49 dB, the literature model with multiple filters and a single polynomial function (F+P1) had an NMSE of -34.32 dB, and the proposed model (Proposed F+P+) reported an NMSE of -36.53 dB. For  $M=2$ , F1P+ obtained an NMSE of -36.84 dB, Previous F+P+ obtained an NMSE of -36.85 dB, F+P1 obtained an NMSE of -34.42 dB, and Proposed F+P+ obtained an NMSE of -36.86 dB.

**Keywords**—behavioral modeling, power amplifier, efficiency

## I. INTRODUCTION

Currently, there is a high demand for wireless communication, data processing and distribution due to the speed of increase in quality and capacity of the electronic devices. Power Amplifiers (PA) have become essential in this context, by increasing the power of the input signal and improving the output signal. However, some problems in its operation occur, such as distortions in the output signal, due to its non-linearity, compromising its efficiency [1].

Artificial neural network and Volterra series are some of the tools to develop a behavioral model of PA [2]. An alternative to reduce the complexity of Volterra-based models is to use block-oriented models based on digital filters and one-dimensional polynomials [3]. The aim of this paper is to propose a new behavioral model for the PA and relate the number of filters and polynomial functions with operation quality. In addition, the operation and accuracy of the introduced model will be compared to three literature models.

Section II introduces the Literature Models and Section III introduces the Proposed Model. Each model has a constitutive

equation and a detailed operation illustrated in a block diagram. Then, Section IV introduces the Experimental Validation, Section V describes the result from the comparison of the models and Section VI presents Conclusions.

## II. LITERATURE MODELS

### A. F1P+ Model

The Literature Model I [3] has a single filter and multiple polynomial functions (F1P+). Its constitutive equation is described by:

$$\tilde{y}(n) = \left[ \sum_{m=0}^M \tilde{a}_{m+1} \tilde{x}(n-m) \right] \times \left[ \sum_{p=1}^P \sum_{m=0}^M \tilde{b}_{pm} |\tilde{x}(n-m)|^{p-1} \right], \quad (1)$$

where  $\tilde{x}(n)$  is input signal at sample  $n$  and  $\tilde{y}(n)$  is output signal at sample  $n$ ,  $M$  is the memory depth,  $P$  is the polynomial order and  $\tilde{a}_{m+1}$  and  $\tilde{b}_{pm}$  are the model complex-valued coefficients. The block diagram of the model is shown in fig 1.

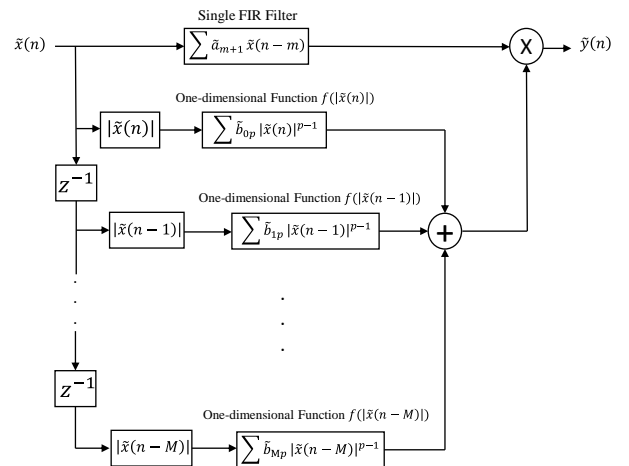


Fig. 1. Block diagram of the Literature Model I called F1P+

This model operation is based on multiplying the single FIR filter with the sum of multiple polynomial functions. The parameter  $\tilde{a}_{m_1+1}$  is identified with non-linear optimization tools [4]. After that, the method of least squares can be used to extract the coefficient  $\tilde{b}_{pm}$  [5].

### B. Previous F+P+ Model

The Literature Model II [3] has multiple filters and multiple polynomial functions (previous F+P+). Its constitutive equation is described by:

$$\tilde{y}(n) = \sum_{m_1=0}^M \left[ \sum_{m_2=0}^M \tilde{a}_{m_1+1, m_2+1} \tilde{x}(n - m_2) \right] \times \left[ \sum_{p=1}^P \tilde{b}_{m_1 p} |\tilde{x}(n - m_1)|^{p-1} \right], \quad (2)$$

where  $\tilde{x}(n)$  is input signal at sample  $n$  and  $\tilde{y}(n)$  is output signal at sample  $n$ ,  $M$  is the memory depth,  $P$  is the polynomial order and  $\tilde{a}_{m_1+1, m_2+1}$  and  $\tilde{b}_{m_1 p}$  are the model complex-valued coefficients. The block diagram of the model is shown in fig 2.

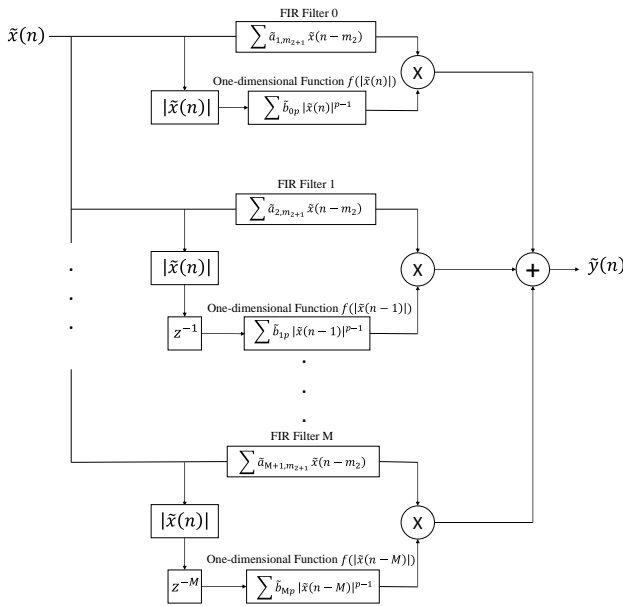


Fig. 2. Block diagram of the Literature Model II called Previous F+P+

This model operation is based on multiplying multiple FIR filters with the corresponding polynomial functions and then summing the individual results. The parameter  $\tilde{a}_{m_1+1, m_2+1}$  is identified with non-linear optimization tools [4]. After that, the method of least squares can be used to extract the coefficient  $\tilde{b}_{m_1 p}$  [5].

### C. F+P1 Model

The Literature Model III [6] has multiple filters and a single polynomial function (F+P1). Its constitutive equation is described by:

$$\tilde{y}(n) = \sum_{m_2=0}^M \left\{ \sum_{m_1=0}^M \tilde{a}_{m_1, m_2} \left[ \sum_{p=1}^P \tilde{b}_p |\tilde{x}(n - m_1)|^{p-1} \right] \right\} \times \tilde{x}(n - m_2), \quad (3)$$

where  $\tilde{x}(n)$  is input signal at sample  $n$  and  $\tilde{y}(n)$  is output signal at sample  $n$ ,  $M$  is the memory depth,  $P$  is the polynomial order and  $\tilde{a}_{m_1, m_2}$  and  $\tilde{b}_p$  are the model complex-valued coefficients. The block diagram of the model is shown in fig 3.

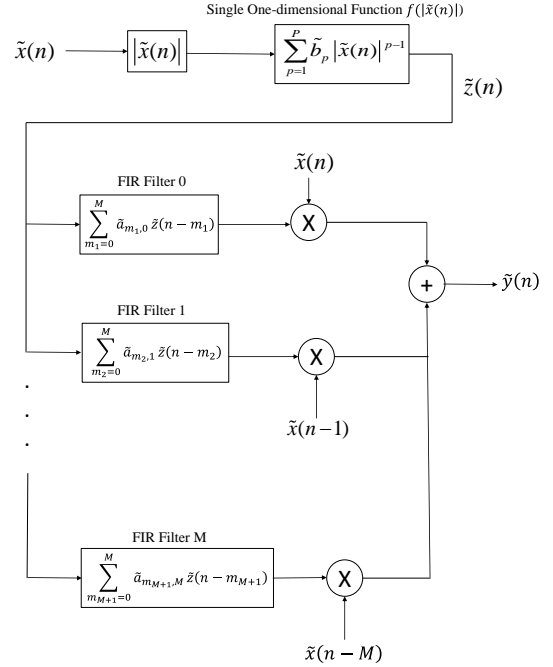


Fig. 3. Block diagram of the Literature Model III called F+P1

This model operation is based on interactions of the single polynomial function with multiple FIR filters and multiplying with corresponding input sample. Then, sum of all the operations. The parameter  $\tilde{b}_p$  is identified with non-linear optimization tools [4]. After that, the method of least squares can be used to extract the coefficient  $\tilde{a}_{m_1, m_2}$  [5].

## III. PROPOSED MODEL

The Proposed Model was created based on the hypothesis that an alternative distribution, with respect to Previous F+P+ Model, of several digital filters and several polynomial functions can be obtained. It has multiple filters and multiple polynomial functions (proposed F+P+). Its constitutive equation is described by:

$$\tilde{y}(n) = \sum_{m_2=0}^M \left\{ \sum_{m_1=0}^M \tilde{a}_{m_1, m_2} \left[ \sum_{p=1}^P \tilde{b}_{p, m_2} |\tilde{x}(n - m_1)|^{p-1} \right] \right\} \times \tilde{x}(n - m_2), \quad (4)$$

where  $\tilde{x}(n)$  is input signal at sample  $n$  and  $\tilde{y}(n)$  is output signal at sample  $n$ ,  $M$  is the memory depth,  $P$  is the polynomial order and  $\tilde{a}_{m_1, m_2}$  and  $\tilde{b}_{p, m_2}$  are the model complex-valued coefficients. The block diagram of the model is shown in fig 4.

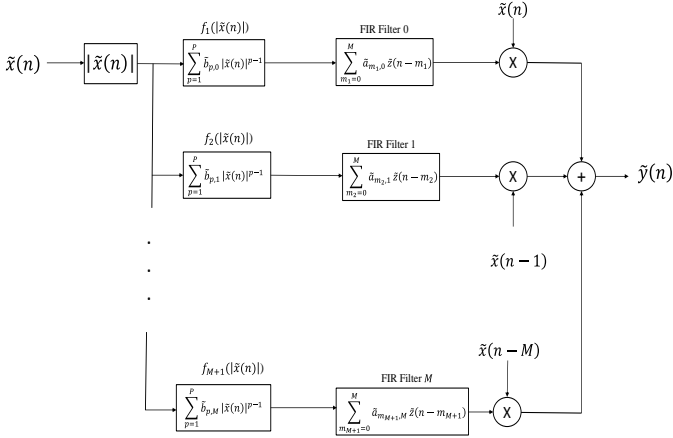


Fig. 4. Block diagram of the Proposed Model called Proposed F+P+

This model operation is based on interactions of multiple polynomial functions with corresponding FIR filters and multiplying with corresponding input sample. Then, sum of all the operations. The parameter  $\tilde{b}_{p, m_2}$  is identified with non-linear optimization tools [4]. After that, the method of least squares can be used to extract the coefficient  $\tilde{a}_{m_1, m_2}$  [5].

#### IV. EXPERIMENTAL VALIDATION

All the Experimental Validation was based on a GaN PA. This PA is driven by a 900 MHz carrier. It has a class AB, and the envelope is a 3GPP WCDMA with a bandwidth of about 3.84 MHz.

There were two different kinds of dataset. Extraction data, which was used to find the values of the coefficients, and the Validation data, which was used to calculate output values and validate model accuracy.

The values of  $P$  and  $M$  are arbitrary, but for the four models the polynomial order was set to  $P = 3$ . In this comparative study, two situations were created, when the memory depth was set equal to  $M = 1$  and  $M = 2$ . For each different  $M$  value, each model was simulated five times.

All the necessary algorithms were implemented in Matlab software using double-precision floating-point arithmetic with the assistance of non-linear optimization from *lsqnonlin* and least squares methods with the “\” command.

For each simulation, exactly five realizations were generated starting with random initial guesses and the dataset of coefficients which produced the smaller modeling error was chosen for the validation step.

In other to validate the accuracy of each one of the models, the normalized mean square error (NMSE) metric was implemented, described by:

$$NMSE = 10 \log_{10} \frac{\sum_{n=1}^N |y_{real}(n) - y_{sim}(n)|^2}{\sum_{n=1}^N |y_{real}(n)|^2}, \quad (5)$$

where  $y_{real}(n)$  is output signal at sample  $n$  from validation data and  $y_{sim}(n)$  is output signal at the sample  $n$  from the behavior simulation.

#### V. RESULTS

After all the simulations, NMSE values for each model and different  $M$  values were found and compared in Table I.

TABLE I. NMSE (dB) VALUES OBTAINED FOR EACH ONE OF THE IMPLEMENTED MODELS

	F1P+	Previous F+P+	F+P1	Proposed F+P+
$M = 1$	-36.49	-36.49	-34.32	-36.53
$M = 2$	-36.84	-36.85	-34.42	-36.86

According to the Table I, the Proposed F+P+ got the best accuracy, when  $M = 1$ , with -36.53 dB and when  $M = 2$ , with -36.86 dB. Those results can be seen in fig. 5, related to  $M = 1$ , and fig. 6 related to  $M = 2$ .

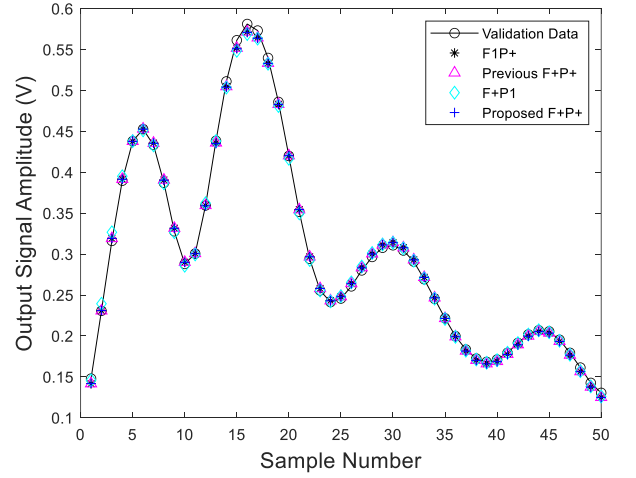


Fig. 5. Output Signal Amplitude versus Sample Number: when  $M = 1$

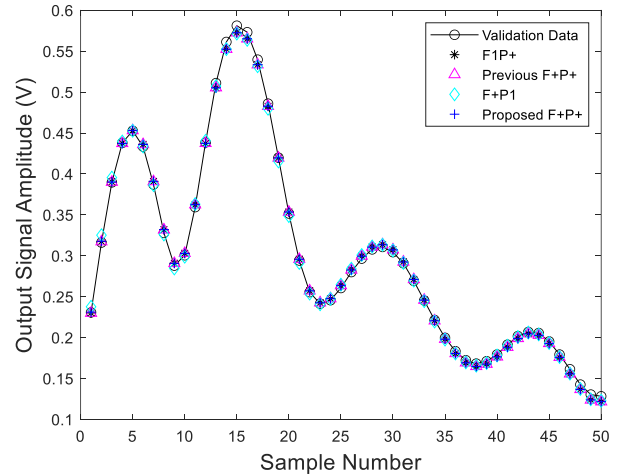


Fig. 6. Output Signal Amplitude versus Sample Number: when  $M = 2$

Although the Proposed F+P+ has the better performance, according to Table I, other models have a similar behavior and output signal amplitude. Figs. 7 and 8 describe the accuracy behavior when comparing the models through a measured AM-AM (Amplitude Modulation – Amplitude Modulation).

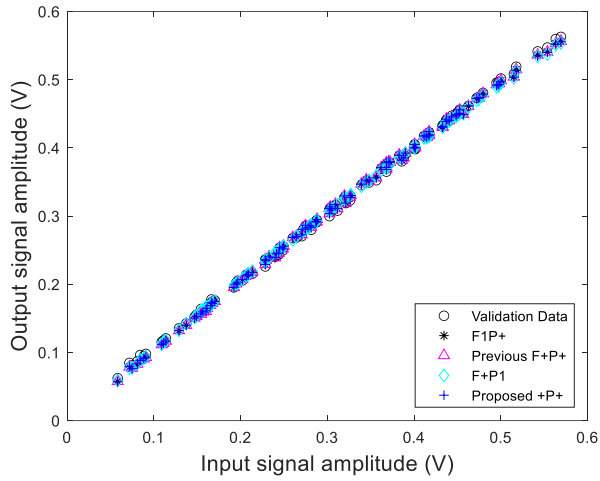


Fig. 7. AM-AM characteristic: when  $M = 1$

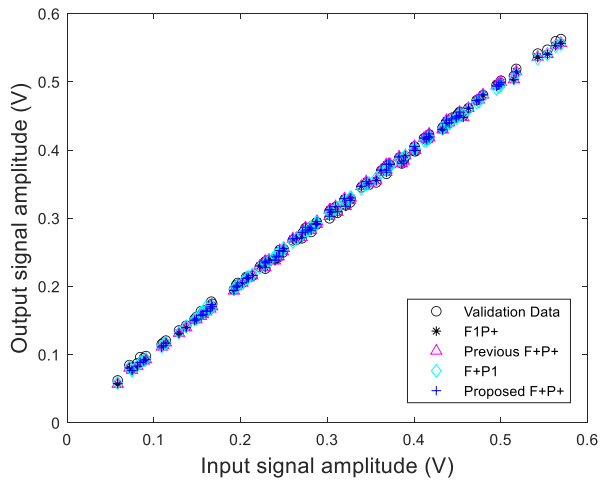


Fig. 8. AM-AM characteristic: when  $M = 2$

## VI. CONCLUSIONS

The models discussed in Sections II have different characteristics and behaviors. Compared F1P+ with Previous F+P+, and F+P1 with Proposed F+P+, it is possible to notice that

operation is the same, but the difference is the number of the functions and filters.

The values of  $P$  and  $M$  are arbitrary. In this paper the polynomial order was set to  $P = 3$ . The memory depth was set  $M = 1$  and  $M = 2$ , so two comparison situations were created. All the necessary algorithms were implemented in Matlab software.

The Experimental Validation step was based on a PA GaN and comparing the NMSE value of each model and case.

For  $M = 1$ , NMSE values for F1P+, Previous F+P+, F+P1 and Proposed F+P+ were -36.49 dB, -36.49 dB, -34.32 dB and -36.53 dB, respectively. For  $M = 2$ , NMSE values for F1P+, Previous F+P+, F+P1 and Proposed F+P+ were -36.84 dB, -36.85 dB, -34.42 dB and -36.86 dB, respectively. In both cases the Proposed Model (Proposed F+P+) has the best accuracy.

However, according to the comparative illustrative curves, it was noticed that all models have similar behaviors.

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