Quasi-Periodic Steady-State and Periodic AC Applied for One Large and One Small Signal Tone

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***Abstract*—Simulations are an important part in the field of electronics, allowing the study of circuit behavior. Two-tone analysis is commonly used in non-linear situations, and in this segment Periodic Alternating Current Analysis (PAC) and Quasi-Periodic Steady-State Analysis (QPSS) stand out. The first method requires knowledge of voltages and currents at all time, in addition to using larger matrices in the simulation and linearization. The second method, on the other hand, can be simulated with sampled information. Thus, the present work explores the performance of these different analyses with the imposition of small and large signals for two distinct frequencies. The study was carried out through programming with the Python language, using the libraries that it provides. Throughout the work, through graphical analysis, the possibility of applying small and large signals in different scenarios using the QPSS is explored, considering two distinct frequencies. The results for QPSS confirm the possibility of applying the analysis of small and large signals by varying the number of harmonics used for each frequency. Furthermore, tests with the PAC method show that, differently from the QPSS, the frequency is independent for the imposition of small signals, as the results obtained will be similar.**

***Index Terms*—Quasi-Periodic Steady-State, Circuit Linearization, Periodic Alternate Current**

I. Introduction

Simulations are fundamental for the study and development of electronic devices, as they allow observing the behavior of the circuit in its daily application and in other extreme cases, without the need for physical implementations and simulations of environments, reducing production costs. Furthermore, it is possible to perform optimizations on the schematic without the need for full human attention. However, with the increase in the complexity of the circuit and the frequencies that act on it, the simulation becomes more computationally expensive, since there is a greater number of unknowns to be solved and a longer operation time. As a result, there is a demand for methods to make more complex simulations viable [1].

The two-tone analysis is often used in non-linear circuits, since two excitation signals with different frequencies generate the inter-modulation products. Sometimes, an amplitude of large signals for the first frequency and an amplitude of small signals for the second frequency are considered in the analysis. In radio-frequency circuits, for example, a sinusoidal signal can be considered at the input, mixed with the signal of a local oscillator with much smaller amplitude through a mixer [2].

Thus, techniques for analyzing circuits were developed that aim to obtain the response in two tones, where the Periodic Alternating Current Analysis (PAC) and the Quasi-Periodic Steady-State Analysis (QPSS) stand out. In the PAC, voltages and currents need to be known at all times of the simulation [3], the circuit is linearized around the operation in large signals and harmonics are not considered for the tone in small signals. In QPSS, the linearization of any amplitude that composes the input signal is not necessary, with voltages and currents being calculated only at equally spaced sampled instants, whose amplitudes depend on multiple instants of the smallest period between the input frequencies [3].

In this scenario, this work aims to explore procedures for PAC and QPSS simulations in non-linear circuits. These are fed by two sinusoidal sources with frequencies ω_1 and ω_2, testing the possibilities of imposing small and large signals for each of the frequencies.

The work is organized in order to initially discuss the state of the art in Section I, with the presentation of the importance of circuit simulation and the methods to be approached in the two tone analysis. In Section II, the PAC method is presented in more detail, explaining its logic and operating conditions. In Section III, the QPSS method is discussed, along with its elaboration and specifications. Section IV presents the non-linear circuit in which the simulations will be carried out, together with the results of the methods discussed in the work, while Section V discusses the conclusions about the results and topics covered.

II. Periodic AC Analysis

The PAC consists in the application of the superposition of the non-linear and linear analyses of the inputs in two tones of the circuit [4], according to

$$v_{t2tones} = v_{nl} + v_l, \tag{1}$$

where v_{nl} is the result of the non-linear analysis using Harmonic Balance (HB), which must be done at the highest amplitude tone [4], and v_l is the portion resulting from the linearization of the circuit.

When assuming that one of the tones is of small signals, the PAC imposes that this tone does not present harmonics, computing the respective harmonics to each frequency, $m\omega_1 + n\omega_2$, where $m = 0, \pm 1, \pm 2, \ldots$, and $n = \pm 1$ [2].

The linearization of the non-linear elements of the circuit is done by substituting their characteristic equations by linear equations. This results in the gain vector g(t) with dimensions $k = 2{*}(K_1{+}1)$, varying in time as in HB. In the vector, K_1 is the number of harmonics considered in the non-linear analysis, which must be twice as large as the number of harmonics, K, considered in the linearized circuit analysis. This consideration must be made so that it is possible to elaborate a completely filled quadratic conductance matrix G, of dimension k [4].

Therefore, it is possible to rewrite the equation of the non-linear element as

$$I_{lin} = \begin{bmatrix} g_0 & 0 & \frac{g_2}{2} & \frac{-g_3}{2} & \cdots & \frac{-g_{k-3}}{2} \\ 0 & g_0 & \frac{g_1}{2} & \frac{g_4}{2} & \cdots & \frac{g_{k-2}}{2} \\ \frac{g_2}{2} & \frac{g_1}{2} & g_0 & \frac{-g_1}{2} & \cdots & \frac{-g_{k-5}}{2} \\ \frac{-g_3}{2} & \frac{g_4}{2} & \frac{-g_1}{2} & g_0 & \cdots & \frac{g_{k-4}}{2} \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ \frac{-g_{k-3}}{2} & \frac{g_{k-2}}{2} & \frac{-g_{k-5}}{2} & \frac{g_{k-4}}{2} & 0 & g_0 \end{bmatrix} \begin{bmatrix} v_{k-2}\sin[(\omega_2 - K\omega_1)t] \\ v_{k-1}\cos[(\omega_2 - K\omega_1)t] \\ \vdots \\ v_4\sin[(\omega_2 - 1\omega_1)t] \\ v_5\cos[(\omega_2 - 1\omega_1)t] \\ v_0\sin(\omega_2 t) \\ v_1\cos(\omega_2 t) \\ v_2\sin[(\omega_2 + 1\omega_1)t] \\ v_3\cos[(\omega_2 + 1\omega_1)t] \\ \vdots \\ v_{k-4}\sin[(\omega_2 + K\omega_1)t] \\ v_{k-3}\cos[(\omega_2 + K\omega_1)t] \end{bmatrix}. \quad (2)$$

It is also necessary to rewrite the Ω matrix, now considering the frequency of the second tone together with that of the first tone, according to (1) to obtain:

$$\Omega_{2tones} = \begin{bmatrix} 0 & (\omega_2 - K\omega_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -(\omega_2 - K\omega_1) & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & 0 & (\omega_2 - \omega_1) & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & -(\omega_2 - \omega_1) & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & \omega_2 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & -\omega_2 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & 0 & 0 & (\omega_2 + \omega_1) & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & 0 & -(\omega_2 + \omega_1) & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & (\omega_2 + K\omega_1) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(\omega_2 + K\omega_1) & 0 & 0 \end{bmatrix}. \quad (3)$$

In Fig. 1, a flowchart is presented in order to summarize the PAC elaboration process.

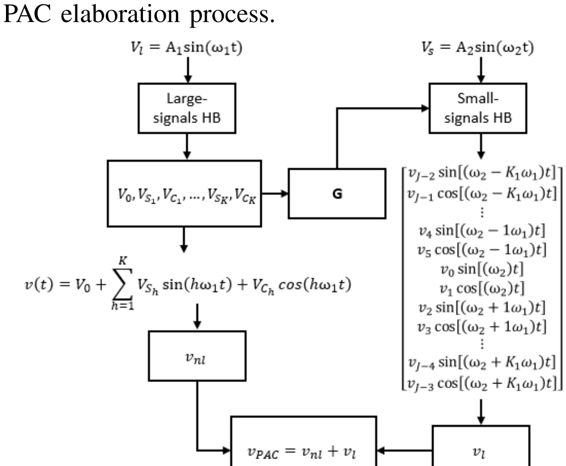

Fig. 1: PAC Flowchart.

III. Quasi-Periodic Steady-State Analysis

In circuits excited by two tones, with $\omega_1 < \omega_2$, where the system is in steady state, it can be assumed that the sampling of a nodal voltage to be calculated in the circuit is periodic [5]. Given these conditions and the fact that the sampling can be represented by a Fourier series truncated in a few terms, it is possible to develop the QPSS.

Each sampling point of the fundamental period is performed at the instants $\tau_1, \tau_2, \ldots, \tau_S$, where $S = 2 * K + 1$, there K depends of the number of considered harmonics of the lowest frequency. Thus, a nodal voltage at each sampled point can be written according to

$$v_n(\tau_s) = V_0 + \sum_{h=1}^{K} V_{S_h}\sin(h\omega_1\tau_S) + V_{C_h}\cos(h\omega_1\tau_S), \quad (4)$$

where V_0, V_{S_h} and V_{C_h} are spectral constants given by QPSS, with the numbers of constants dependent of S. Analysing (4), it is possible to see the need for a relationship between the instants of each sampled point, that is, an operation that allows advancing each nodal voltage at the sampled point in a period of the highest frequency, T. Thereby, the operation must transform $v_n(\tau_1), \ldots, v_n(\tau_S)$ in $v_n(\tau_1 + T), \ldots, v_n(\tau_S + T)$.

This operation can be performed using the delay matrix [5], where the matrix Γ^{-1}, similar to Inverse Fourier Transform is used. The matrix Γ^{-1} computes the coefficients of (4) in the frequency domain, according to

$$\Gamma^{-1} \begin{bmatrix} V_0 \\ V_{S_1} \\ V_{C_1} \\ \vdots \\ V_{S_K} \\ V_{C_K} \end{bmatrix} = \begin{bmatrix} v_n(\tau_1) \\ v_n(\tau_2) \\ v_n(\tau_3) \\ \vdots \\ v_n(\tau_S) \end{bmatrix}. \quad (5)$$

Therefore, for the values to be shifted by a period T, it is necessary to rewrite $\Gamma^{-1}(T)$ as

$$\Gamma^{-1}(T) = \begin{bmatrix} 1 & \sin(2\pi 1(\tau_1{+}T)) & \cos(2\pi 1(\tau_1{+}T)) & \cdots & \sin(2\pi K(\tau_1{+}T)) & \cos(2\pi K(\tau_1{+}T)) \\ 1 & \sin(2\pi 1(\tau_2{+}T)) & \cos(2\pi 1(\tau_2{+}T)) & \cdots & \sin(2\pi K(\tau_2{+}T)) & \cos(2\pi K(\tau_2{+}T)) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \sin(2\pi 1(\tau_S{+}T)) & \cos(2\pi 1(\tau_S{+}T)) & \cdots & \sin(2\pi K(\tau_S{+}T)) & \cos(2\pi K(\tau_S{+}T)) \end{bmatrix}. \quad (6)$$

Thus, the time domain elements can be shifted by first applying Γ to transform the time domain elements to frequency, and then applying $\Gamma^{-1}(T)$ and the delay matrix, as

$$\begin{bmatrix} v_n(\tau_1 + T) \\ v_n(\tau_2 + T) \\ \vdots \\ v_n(\tau_S + T) \end{bmatrix} = \Gamma^{-1}(T)\Gamma \begin{bmatrix} v_n(\tau_1) \\ v_n(\tau_2) \\ \vdots \\ v_n(\tau_S) \end{bmatrix}, \quad (7)$$

and the delay matrix can be represented by:

$$D(T) = \Gamma^{-1}(T)\Gamma, \quad (8)$$

where $D \in R^{SxS}$. So that they can be used in any node, since it only depends on ω_1, τ and T. That is, the number of harmonics considered for the fundamental frequency interferes with the number of displacements made by D, thus also interfering with its dimension.

In (7), to ensure that the circuit is in steady state according to the precondition established for the operation of the QPSS, the left side can be computed through transients of size T, where the number of points to be simulated depends on the number of harmonics to be used for the highest frequency. Then the number of points to be considered in each transient is $\eta = 2*K_1+1$, because that depends on the highest frequency.

The QPSS allows both tones present in the analysis to be large-signal, and their harmonics can be adjusted independently. The QPSS construction flowchart is shown in Fig. 2.

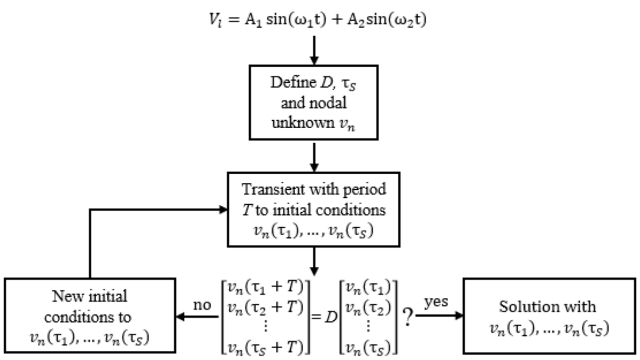

Fig. 2: QPSS flowchart.

IV. Results

To compare the described methods, the circuit shown in Fig. 3 is used. The circuit is an approximation of a power amplifier with envelope tracking [6] architecture .

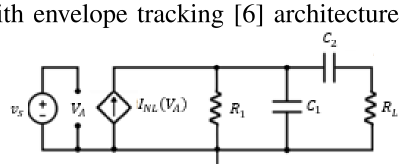

Fig. 3: Test circuit schematic.

In Fig. 3, the fixed parameters are R_1=1 $k\Omega$, R_L=50 Ω, C_1= 10 pF and C_2=1 μF. The V_s source is a two-tone voltage source, with different amplitudes and frequencies, where f_1=1 GHz for the first tone, and f_2=1.01 GHz for the second tone. The non-linear current source I_{NL} depends on the voltage V_A, and its equation is given by

$$I_{NL} = \frac{I_{sat}(V_A)}{(1 + \dfrac{V_{sat}}{|V_A|})^s} \quad (9)$$

where the current saturation I_{sat}=0.1 A, the voltage saturation V_{sat}=1.8 V and the damping factor is s=5.

The simulations were performed in a Python programming environment, using the Numpy, Scipy and Matplotlib libraries. These libraries were explored in the performance of matrix operations, in the use of the "fsolve" function to find the solution of non-linear equations and in the elaboration of graphs with the responses of the simulations. For the tests, it was sought to vary the amplitudes A_1 for the first tone and A_2 for the second tone in order to impose one large signal and one small signal for the two frequencies handled by each method. In the PAC, the tone to be considered as small signals is the one that receives the smallest amplitude.

In QPSS, small signal enforcement takes place differently. For a tone to be considered small-signal, the harmonic in question must be unity, while the number of harmonics to be considered for the large-signal tone must be chosen arbitrarily. Therefore, when considering small signals for the lower frequency tone, K = 1 and K_1 = 100, and when considering small signals for the higher frequency tone, K = 4 and K_1 = 1.

Two cases were analysed to validate two results of the QPSS and PAC, being made together the waveform resulting from a transient analysis in one cycle of the circuit.

In a first case, it is expected to validate the imposition of small signals for the slowest frequency of the circuit. For this, in QPSS, amplitudes that distort the carrier frequency and keep the envelope as a pure sinusoid are chosen, and thus the values A_1=5 V and A_2=1 V were selected for the amplitudes.

In the second case, it is necessary to validate the imposition of small signals for the fastest frequency, that is, a distorted curve for the envelope, in which the carrier wave is a pure sinusoid. Therefore, the value of 1 V was chosen for both amplitudes.

In both cases, to validate the PAC, it was considered 1 V to large-signal and 0.01 V to the small-signal and it was changed the frequency that receives these amplitudes in each situation.

A. Imposition of small signals to the lowest frequency

In Fig. 4 the capacitor C_1 voltage for the transient simulation and the execution of the QPSS for small signals are presented, applying lower (red dots) and higher frequency (yellow dots). The bottom graph presents a macroscopic view of the waveforms, while the top graph shows in more detail the voltage behavior on a smaller scale.

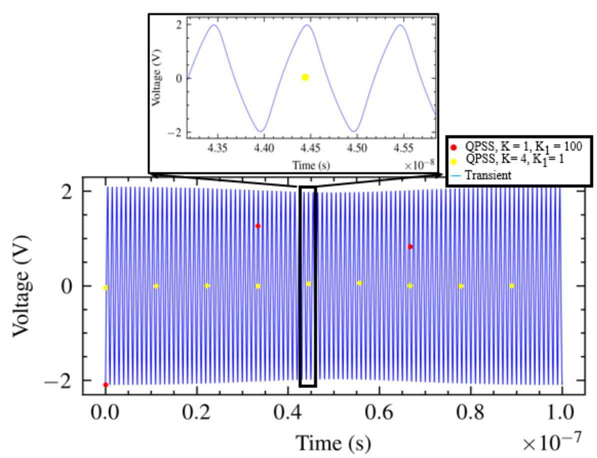

Fig. 4: Waveforms with A_1=5 V and A_2=1 V with small signal applied in the lowest frequency.

From Fig. 4, it can be seen that the small signals are not valid for the carrier, since it is distorted. However, there is validity to small signals to the envelope since it presents a pure sinusoidal. Therefore, this analysis should also be valid for the QPSS results, which is proven in Fig. 4: the imposition of small signals to the envelope is accurate compared to the transient (yellow dots), while the imposition of small signals to the carrier results in inaccurate dots (red dots).

The Fig. 5 shows the voltage across capacitor C_1 when using the PAC method with small imposed signals for the slowest frequency.

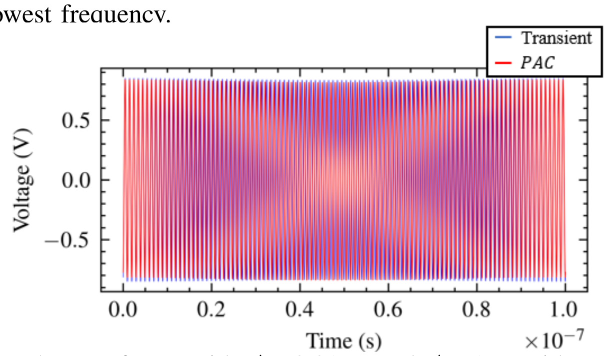

Fig. 5: Waveforms with A_1=0.01 V and A_2=1 V with small signal applied in the lowest frequency.

When considering small signals for the slowest frequency, similar results to the ones obtained when considering small signals for higher frequencies are observed. Therefore, for the PAC, it does not depend on which frequency is being used as small signals, since similar values are obtained.

B. Imposition of small signals to the highest frequency

In Fig. 6 the capacitor C_1 voltage for the second case to be analysed is presented. The bottom graph presents a macroscopic view of the waveforms, while the top graph shows in more detail the voltage behavior on a smaller scale.

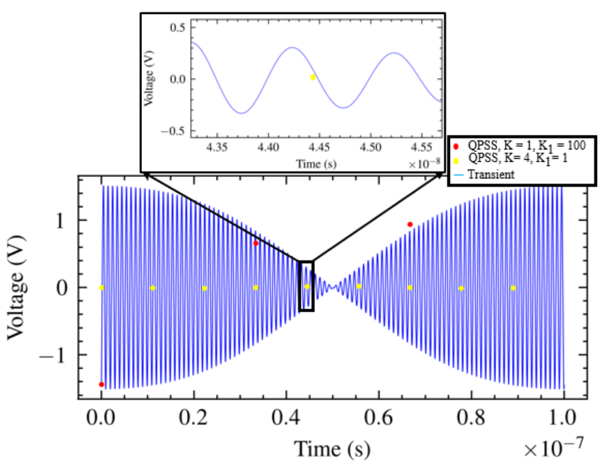

Fig. 6: Waveforms with A_1=5 V and A_2=1 V with small signal applied in the highest frequency.

It is possible to observe in the transient that the envelope is distorted, and the carrier is a pure sinusoidal, that is, the validity of small signals is only for the carrier. Thus, using QPSS, the points for imposing small signals to the envelope must be imprecise, and the points for imposing small signals to the carrier must be accurate, which is in fact observed in Fig. 6.

The Fig. 7 shows the voltage across capacitor C_1 using the PAC, and considering small signals for the highest frequency.

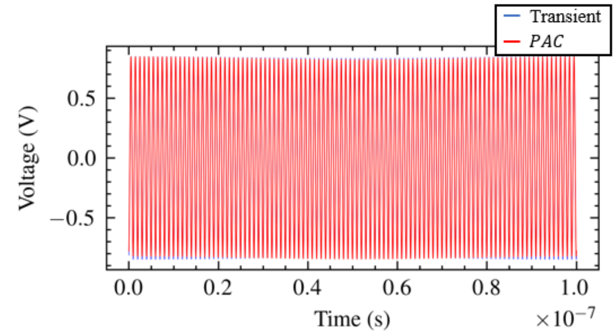

Fig. 7: Waveforms with A_1=5 V and A_2=1 V with small signal applied in the highest frequency.

It is noticeable that the PAC results are very similar to those obtained by the transient, validating the case of imposition of small signals for the highest frequency.

V. Conclusions

As proposed in this work, it was possible to verify the possibility of imposing small and large signals for different tones using QPSS and PAC. For QPSS, changing which tone receives small signals is done by varying the number of harmonics considered at each frequency. For the PAC, this change is made by changing which frequency is considered the greatest amplitude, and the imposition of small signals in one tone is obligatory.

The QPSS takes into account the distortions caused by the intermodulation of the signals, resulting in the inaccuracy of the points depending on the situation that applies small signals to the circuit and the way of this imposition differs depending of the considered tone. Therefore, it must be considered in the analysis which of the two frequencies the small signals will be imposed. Also, the imposition of small signals in one tone is not necessary.

However, in PAC, changing the frequency that receives small signals results in similar values, thus the variation is irrelevant to the method.

Acknowledgment

The authors would like to acknowledge the financial support provided by National Council for Scientific and Technological Development (CNPq) and Fundação Araucária de Apoio ao Desenvolvimento Técnico Científico e Tecnológico do Estado do Paraná under the Program PIBIS UFPR 2021.

References

[1] G. D. Vendelin, A. M. Pavio, M. Rudolph, U. L. Rohde, Microwave Circuit Design Using Linear and Non-linear Techniques. Wiley, 2021.

[2] MASS, S., Non-linear Microwave and RF Circuits, Second Edition ed. www.artechhouse.com, 2003.

[3] Chen, J.; Feng, D.; Philips, J.; Kundert, K. Simulation and Modeling of Intermodulation Distorcion in Communication Circuits. Cadence Design Systems, San Jose, California, USA, 1999.

[4] Kundert, K. S.; White, J.; S-Vicentelli, A. A Mixed Frequency-Time Approach for Distortion Analysis of Switching Filter Circuits. IEEE Journal of Solid-State Circuits, Vol. 24, No. 2. April, 1989, pp. 443–451.

[5] MOURA, D. M. S.; NATALINO, C. G. ; LIMA, E. G. . Linearização de Circuitos Eletrônicos em Torno de Regime Permanente a Zero, Um e Dois Tons. In: Seminários de Microeletrônica do Paraná, 2019, Curitiba. Anais do SeMicro-PR, 2019

[6] SCHUARTZ, L., SANTOS, E. L., LEITE, B., MARIANO, A. A., LIMA, E. G. Envelope methods Applied to simulation of Power Amplifiers with Efficiency Enhancement Techniques. EMICROSIM, 2018. Proceedings of EMICROSIM 2018.